Open Problems from CCCG 2014

Sue Whitesides *

Abstract

This report provides the problems posed by the participants at the open problem session of the 26^{th} Canadian Conference on Computational Geometry.

This well-attended session was held Tuesday, August 12, 2014, as a scheduled session of the conference. Six participants presented a total of seven problems. All presenters kindly agreed to provide written versions of their problems, including references and attributions. The problems appear in the sections below. The references appear at the end. The text is essentially the same, modulo minor editing, as the text provided by the presenters. This material is not refereed.

1 Guarding Orthogonal Terrains

presented by: Giovanni Viglietta[1]

Partition the plane into finitely many (possibly unbounded) orthogonal polygons, and extrude them in 3D, obtaining a set of "orthogonal skyscrapers" of different heights. Let n be the total number of vertices of the orthogonal polygons. We ask to find the minimum number (as a function of n) of vertex guards for the terrain induced by the skyscrapers. In other words, we seek to select a minimum number of "guards" among the vertices of the skyscrapers such that each point in 3-space lying "above" some skyscraper is visible to some guard, where lines of sight must not intersect a skyscraper's top face or a side face.

The best known lower bound is given by a row of k equal cuboidal skyscrapers, where $n = 8k$. In this case $k + 1$ vertex guards are needed, which yields a lower bound of $(n/8) + 1$ vertex guards. We conjecture $n/8 + O(1)$ guards to be sufficient for all orthogonal terrains on n vertices (observe that an L-shaped skyscraper on 12 vertices needs three guards). To our knowledge, the problem is open even in the case of a single "tower" made of nested orthogonal prisms of increasing height, or a single "well".

For background, see [1].

*Professor, Department of Computer Science, U. of Victoria, Canada; email: sue@uvic.ca

[1]Postdoctoral Fellow, U. of Ottawa and School of Computer Science, Carleton U., Canada; email: viglietta@gmail.com

2 Flows on Terrains

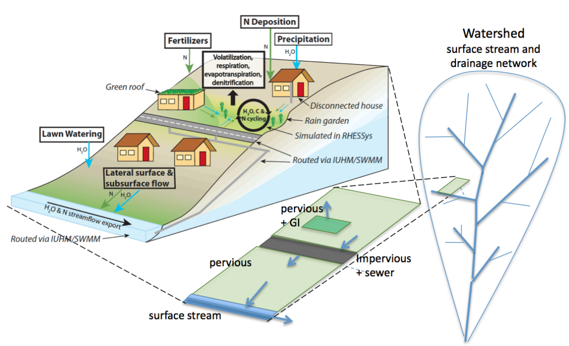
presented by: Jack Snoeyink[2]

What local actions can make a general difference for flow of water, nutrients, and pollutants in a terrain? This is more of an open application area for computational geometry techniques than an open problem.

Consider a real-world terrain with patches having different soil types (e.g., different absorbency properties) together with a network of streams, house gutters, parking lot drains, and underground sewers. There are rain gauges reporting rainfall in cm/hr at some points and flow meters reporting liters/min profiles on some waterways. (These are increasingly common in the "internet of things.")

If we model a rainfall, do we see the measured flows? If not, can we suggest where our information about the flow network is incomplete or inaccurate? If we don't like, say, the surge of flow in the sewers from a rainfall, can we suggest where rain gardens could most effectively delay the flow? At what scale should these questions be asked based on the sensors we have?

There are many simulations that are used [2, 3], but the ideas of computational geometry (like continuous Dijkstra for paths in weighted regions [4], or partitioning terrain into catchments and capturing flow in equilibrium [5]) can be used to preprocess the terrain for more efficient exploration of modifications that would produce the observed or desired flow profiles.



[2]Professor, Dept. of Computer Science, U. North Carolina; email: snoeyink@cs.unc.edu

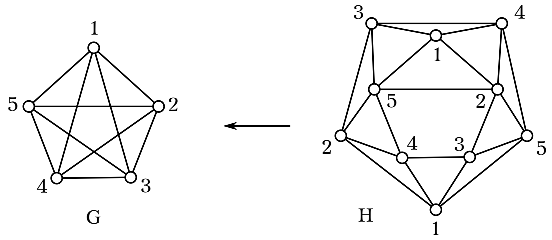


Figure 1: $H \simeq K_5$ and its finite planar emulator H.

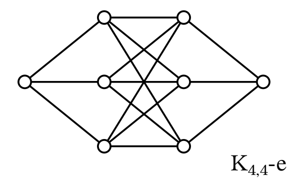


Figure 2: $K_{4,4} - e$, one of the minor-minimal obstructions for the projective plane, where the existence of a finite planar emulator is open.

3 Finding a Shared Delaunay Triangle in Linear Time

presented by: Michael Biro[3]

Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n points in the plane and s, t be two query points. The problem is to determine, in linear time, whether or not s and t lie in the same face of the Delaunary triangulation of P. The problem was posed by Joseph S. B. Mitchell in personal correspondence.

The problem can be solved trivially in time $O(n \log n)$ by constructing the Delaunay triangulation and performing point location queries. However, constructing the full Delaunay triangulation has a lower bound of $\Omega(n \log n)$ so this approach cannot be used to determine the answer in linear time. Thus the question is asking, in essence, if we can quickly find local information about a Delaunay triangulation without first having to construct the entire triangulation.

One reason to expect the answer to be affirmative is that the dual question of determining if s and t lie in the same face of the Voronoi diagram of P is trivial to answer in time $O(n)$: simply find the nearest neighbors of s and t, respectively. The two points s and t share nearest neighbors if and only if they are in the same face of the Voronoi diagram of P.

Jack Snoeyink proposed a linear-time solution by lifting the set P to a paraboloid in 3D and locating the lifted points s and t on faces of the convex hull.

4 Finite Planar Emulators

presented by: Martin Derka[4]

A graph G has a finite *planar emulator* H if H is a planar graph and there is a graph homomorphism $\varphi : V(H) \to V(G)$ where φ is locally surjective, i.e. for every vertex $v \in V(H)$, the neighbours of v in H are mapped surjectively onto the neighbours of $\varphi(v)$ in G. We also say that such a G is *planar-emulable*. If we insist on φ being locally bijective, we get H a *planar cover*.

The concept of planar emulators was proposed in 1985 by M. Fellows [11], and it tightly relates (although it is of independent origin) to the better known *planar cover conjecture* of Negami [12]. Fellows also raised the main question: What is the class of graphs with finite planar emulators?

Soon thereafter, he conjectured that the class of planar-emulable graphs coincides with the class of graphs with finite planar covers (conjectured to be the class of projective graphs by Negami [12]—still open at present). This was later restated as follows:

Conjecture 1 *[M. Fellows, falsified in 2008] A connected graph has a finite planar emulator if and only if it embeds in the projective plane.*

It is known that if a graph embedds in the projective plane, it has a finite planar emulator (which takes form of its finite planar cover). The conjecture fails in the converse. Rieck and Yamashita [13], and Chimani et al. [6] constructed finite planar emulators of all the minor minimal obstructions for the projective plane with the exception of those that have been shown non-planar-emulable already by Fellows (the $K_{3,5}$ and "two disjoint k-graphs" cases), and with the exception of $K_{4,4} - e$. The graph $K_{4,4} - e$ is the only forbidden minor for the projective plane where the existence of a finite planar emulator remains open. For more examples of planar emulators and for some graphs that are not planar-emulable, see [6].

5 Colored Radial Orderings

presented by: Ruy Fabila-Monroy[5]

Let S be a set of n points in general position in the plane. Let p be a point not in S such that $S \cup \{p\}$ is in general position; we call p an *observation point*. A *radial ordering* of S with respect to p is a clockwise circular ordering of the points in S by their angle around p. If every point in S is assigned one of two colors, say red and blue, then a *colored radial ordering* of S with respect to p is a circular clockwise ordering of the colors of the points in S by their angle around p. Let

[3]Visiting Asst. Professor, Mathematics and Statistics, Swarthmore College PA; email: `michael.j.biro@gmail.com`

[4]Ph.D. Student, U. Waterloo; email: `mderka@uwaterloo.ca`

[5]Professor, Dept. of Mathematics, Cinvestav-IPN, Mexico; email: `ruyfabila@math.cinvestav.edu.mx`

$\rho(S)$ be the number of distinct radial orderings of S with respect to every observation point in the plane. Likewise, let $\operatorname{col}\rho(S)$ be the number of distinct colored radial orderings of S with respect to every observation point in the plane. Define the following functions:

$$g(n) := \min\{\rho(S) : S \text{ is a set of } n \text{ points}\}$$
$$g_{\text{col}}(n) := \min\{\rho_{\text{col}}(S) : S \text{ is a set of } m \text{ red and } m \text{ blue points, and } n = 2m\}$$

Open Problems

1. Give a tight asymptotic bound for $g(n)$.
2. Give a tight asymptotic bound for $g_{\text{col}}(n)$.

In [14] it is shown that $g(n) \geq \Omega(n^3)$ and it is conjectured that $g(n) = \Theta(n^4)$. For the colored case in the same paper they showed that $g_{\text{col}}(n) = \Omega(n)$ and gave an example of a set of n red and n blue points with $O(n^2)$ colored radial orderings.

6 Finite Simplicial Complexes

two problems presented by: Tamal Dey[6]

Problem 1: Let $K := K(P)$ be a finite simplicial complex linearly embedded in $\mathbb{R}^d$ with vertex set P. Denote by $f_d^k(K, P)$ the number of k-simplices in K. Consider the following quantity:

$$f_d^k(n) = \max_{K,|P|=n} f_d^k(K, P).$$

What is the correct bound on $f_d^k(n)$ in terms of n, k, d? We know that $f_2^1(n) = \Theta(n)$ because planar graphs have at most $3n$ edges and clearly there are planar graphs with $\Omega(n)$ edges. Next question is: what is f_3^2, that is, how many triangles with a total of n vertices can be linearly embedded in $\mathbb{R}^3$? It was proved in [15] that $f_3^2 = O(n^2)$ and a tight lower bound of $\Omega(n^2)$ exists because cyclic polytopes with n vertices in $\mathbb{R}^3$ have a triangulation with $\Omega(n^2)$ triangles. Actually, the lower bound generalizes, that is,

$$f_d^k(n) = \Omega(n^{\min\{k+1, \lceil \frac{d}{2} \rceil\}})$$

because of the known lower bounds for triangulations of cyclic polytopes in $\mathbb{R}^d$. For example, in $\mathbb{R}^4$, of course there could be all possible $n(n-1)/2 = \Theta(n^2)$ edges, but all possible $\binom{n}{3} = \Theta(n^3)$ triangles cannot be linearly embedded. In fact, the following bound is known [16]

$$f_4^3(n) = O(n^{3-\frac{1}{3}}).$$

conjecture: $f_d^k(n) = \Theta(n^{\min\{k+1, \lceil \frac{d}{2} \rceil\}})$

Problem 2: Let K be a finite simplicial complex linearly emedded in $\mathbb{R}^3$. Let C be any given 1-cycle in K. We are interested in detecting if C is *trivial* in the first homology group, that is, if there is a set of triangles in K whose boundaries when summed over Z_2 give C. This problem can be solved in $O(M(n))$ time by first reducing the boundary matrix of K (triangle-edge matrix) to Echelon form and then reducing a column corresponding to C to see if it becomes an empty column or not. Here $M(n)$ is the matrix multiplication time whose current best bound is $O(n^{2.37..})$.

conjecture: Let K be a finite simplicial complex linearly embedded in $\mathbb{R}^3$ with a total of n simplices. Given a 1-cycle C in K, one can detect if C is trivial in the first homology group (with Z_2 coefficient) in $O(n^2)$ time.

If K is a 2-manifold, the detection can be performed in $O(n)$ time by a simple depth-first walk in K. If K is a 3-manifold, the algorithm in [17] can be modified to accomplish the task in $O(n^2)$ time. The question remains open for general simplicial complexes. Although the conjecture is posed here for K embedded in $\mathbb{R}^3$ and for a 1-cycle C, it can be posed for a finite simplicial complex embedded linearly in $\mathbb{R}^d$ and a given p-cycle C in it.

[6] Professor, Dept. of Computer Science and Engineering, The Ohio State U.; email: `tamaldey@cse.ohio-state.edu`

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