A Combinatorial Bound for Beacon-based Routing in Orthogonal Polygons

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Abstract

Beacon attraction is a movement system whereby a robot (modeled as a point in 2D) moves in a free space so as to always locally minimize its Euclidean distance to an activated beacon (also a point). This results in the robot moving directly towards the beacon when it can, and otherwise sliding along the edge of an obstacle. When a robot can reach the activated beacon by this method, we say that the beacon *attracts* the robot. A *beacon routing* from p to q is a sequence $b_1, b_2, \ldots, b_k$ of beacons such that activating the beacons in order will attract a robot from p to b_1 to $b_2 \ldots$ to b_k, and where a beacon placed at q will attract b_k. A *routing set of beacons* is a set B of beacons such that any two points p, q in the free space have a beacon routing with the intermediate beacons $b_1, b_2, \ldots b_k$ all chosen from B. Here we address the question of "how large must such a B be?" in orthogonal polygons, and show that the answer is "sometimes as large as $\left\lfloor \frac{n-4}{3} \right\rfloor$, but never larger."

1 Background

Beacon attraction has come to the attention of the community recently as a model of greedy geographical routing in dense sensor networks. In this application, each node of the network has a location, and each communication packet knows the location of its destination. Nodes having a packet to deliver forward the packet to their neighbor that is the closest (using Euclidean distance) to the packet's destination [5, 7].

In the abstract geometric setting, the destination point is called a beacon, and the message is considered to be a point (or robot) that greedily moves towards the beacon. The robot, under this motion, may or may not reach the beacon—if it does reach the beacon, we say that the beacon *attracts* the robot's starting point. The attraction relation between points has the flavor of a visibility-type relation, with the interesting twist that it is asymmetric: if point p attracts point q, then it does not follow that point q attracts p. In a series of publications, Biro, Gao, Iwerks, Kostitsyna, and Mitchell have studied various visibility-type questions for beacon attraction, such as computing attraction (and inverse-attraction) regions for points, computing attraction kernels, guarding, and routing [4, 3, 2]. Bae, Shin, and Vigneron addressed beacon-attraction guarding in orthogonal polygons [1].

In beacon-based routing, the goal is to route from a source p to a destination q through a series of intemediate points $b_1, b_2, \ldots b_k$ where b_1 attracts q, b_2 attracts b_1, b_3 attracts b_2, etc., and finally q attracts b_k. The idea is that we activate the beacons $b_1, b_2, \ldots b_k$ individually in turn, and then activate a beacon at q, and we will have attracted p all of the way to q. In the application setting, this corresponds to using greedy geographical routing for each hop in a multi-hop routing for the packet; beacons correspond to *landmark* or *backbone* nodes of the network [8]. Ad-hoc networks (and to some extent, sensor networks) expect to see messages from many different p's to many different q's. Thus it is natural to ask whether we can find some set B of backbone nodes (beacons) such that one can route from *any* p to *any* q using only backbone nodes chosen from B.

We call such a set B a *routing set of beacons*. Biro et al.[3] studied the problem of finding minimum-cardinality routing beacon sets in simple polygons. They established that it is NP-hard to find such a minimum-cardinality B, and that such a B can be as large as, but never exceed, $\left\lfloor \frac{n-2}{2} \right\rfloor$. Biro [2] also conjectured that, in orthogonal polygons, such a B could be as large as, but never exceed, $\left\lfloor \frac{n-4}{4} \right\rfloor$. In this paper, we disprove this conjecture, pinning this maximum minimum size at $\left\lfloor \frac{n-4}{3} \right\rfloor$ instead.

In this paper, we omit many details, lemmas, and proofs due to size constraints. Full details are available in the arXiv preprint [9].

2 Preliminaries

2.1 Routing segments

If p and q are points in a polygon with a beacon routing from p to q, then by a *routing segment* we mean any maximal section of the beacon-routing path during which a point travelling the path is attracted by a single beacon (or by the destination point q). If the beacon routing from p to q starts at p, proceeds to beacon b_1, then to beacon b_2, then to q, then the routing segments are the part from p to b_1, the part from b_1 to b_2, and the part from b_2 to q.

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2.2 Decomposition and neighboring rectangles

Let P be an orthogonal polygon of n vertices in general position; handle special-position instances with the usual perturbation technique. Construct the *vertical decomposition* (or *trapezoidation* [6]) of P by creating a vertical chord from every reflex vertex (see Figure 1).

Because of our restriction to general position, there are $\frac{n-4}{2}$ verticals, decomposing the polygon into $\frac{n-2}{2}$ axis-aligned rectangles. Each such rectangle has between one and four neighboring rectangles. If we form a graph of the neighbor relation on the rectangles, then we have the *dual tree* (or *weak dual*) of the decomposition, as shown in Figure 1.

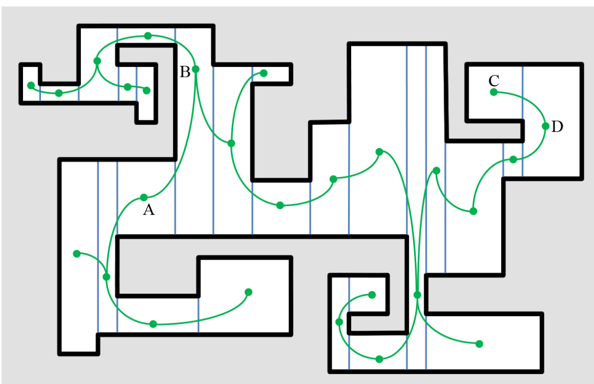

Figure 1: The vertical decomposition of a polygon, with its dual tree.

We classify the different types of neighbors of a rectangle R in 3 primary ways: *left* vs. *right*, depending on the side of R they are on; *top* vs. *bottom*, depending on whether the neighbor and R have the same polygon edge as their top or bottom; and *short* vs. *tall*, depending on whether the neighbor is shorter or taller than R. We combine these classifications: for instance, in Figure 1, A is a short bottom left neighbor of B, and D is a tall top right neighbor of C.

Observation 1 *If a rectangle R is a tall left (or right) neighbor of S, then it is the* only *left (or right, respectively) neighbor of S.*

Observation 2 *If a rectangle R is a short left (or right) neighbor of S, then it is either the* only *left (or right, respectively) neighbor of S (which we call a* solo *neighbor), or there is one other short left (or right, respectively) neighbor of S (in which case we call R a* paired *neighbor of S).*

We generally divide the different cases of a neighboring rectangle's type into into tall, solo, and paired. Figure 2 shows these three types of neighbors.

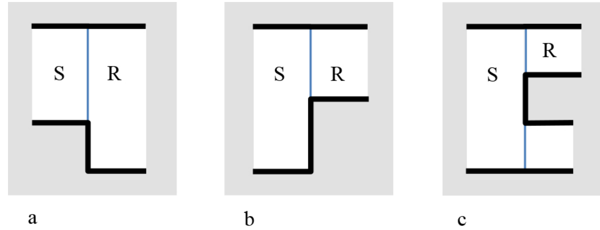

Figure 2: The three types of top right neighbor R of a rectangle S: (a) tall, (b) solo, (c) paired.

2.3 Beacon coverage

If a point p in a polygon attracts a point q, and q attracts p, then we say that p *covers* q. If p covers every point in some region C, then we say that p *covers* C. And if there is a set of points B in the polygon such that for every point q in C, there is a b in B that attracts q, and a b' in B that q attracts, then we say that B covers C. Typically, the point set B will be our set of beacons, and C will be the entire polygon, or a small region of it.

To build a set of beacons we need to know which regions an individual beacon will cover. Fortunately, for our purposes it will mainly suffice to know which rectangles of the decomposition a beacon covers.

First, a beacon b will cover any rectangle of the decomposition it is in. (If b is on a vertical then it will be in two such rectangles.) The lemmas in this section establish some beacon placements that cover rectangles other than their containing rectangles. To save space, in this paper we ignore details about issues of closure that affect the analysis only at reflex vertices. Also, in this section we will omit the proofs of the lemmas but leave the corresponding figures to illustrate definitions and to give the reader a hint at the proofs.

Lemma 1 *If rectangle S is a solo neighbor of rectangle R in the decomposition of a polygon, then any point of R covers S, and any point of S covers R.*

See Figure 3.

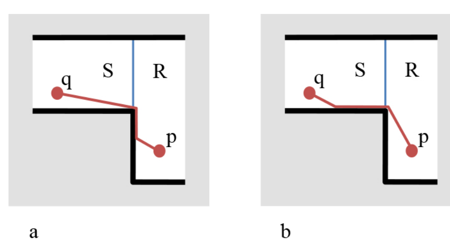

Figure 3: S is a solo neighbor of R. (a) p is attracted into the left wall of R. (b) q is attracted into the bottom wall of S.

Next we look at a rectangle with paired neighbors.

Let R have paired neighbors on the left; we define the *left center* of R as the closed rectangle that is the full width of R and has the vertical span of the polygon edge on the left of R (as illustrated in Figure 4a).

We similarly define the *right center* if R has paired neighbors on the right. See Figure 4.

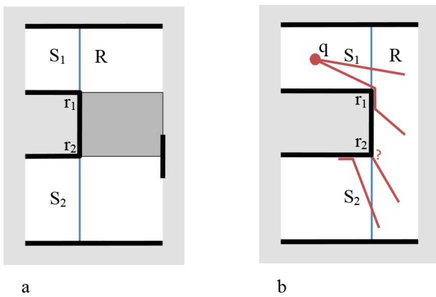

Figure 4: (a) the left center of R is shown shaded. (b) If p is attracted to the left side of R at or above r_1, it proceeds into S_1 (and directly to q). If p is attracted to the left wall of R between r_2 and r_1, it is pulled up the wall and at r_1 will enter S_1 and then will reach q. If p is attracted to the left wall at the point r_2, the behavior is indeterminate. If p is attracted to the left side below r_2, it proceeds into S_2 and does not reach q.

Lemma 2 *If rectangles S_1 and S_2 are paired left (right) neighbors of rectangle R in the decomposition, then any point in the left (right, respectively) center of R covers S_1 and S_2.*

We will mostly be applying Lemma 2 with the point in the center of R being either $r_1 + \varepsilon\hat{x}$ or $r_2 + \varepsilon\hat{x}$, where $\hat{x}$ is the unit vector in the x-direction.

3 Trapping and repair

3.1 Locality

We will call a routing segment *local* if there are (at most) three rectangles of the vertical decomposition whose union contains the segment. We will similarly call a routing path local if all of its segments are local, and a routing beacon set local if it supports a local routing path between every pair of points in the polygon. The routing beacon sets that we construct will all be local.

We let the *local attraction relation* be the attraction relation restricted to those ordered pairs of points (p, q) where p attracts q via a local routing segment.

3.2 Trapped paths

In the inductive step of our proof, we will be removing a few rectangles from the polygon P_k by cutting the polygon along a vertical V of the decomposition. Let C denote the (closed) region that is removed; it will consist of a few rectangles. The (closed) polygon remaining is denoted P_{k+1}. In P_{k+1}, the vertical V is part of the polygon boundary, but in P_k it is not.

To form a beacon set B_k for P_k, we would like to take the beacon set B_{k+1} for P_{k+1} (which inductively exists) and add a few beacons to it. We could use B_{k+1} for routing between pairs of points in P_{k+1} (as a subset of P_k), and then just worry about routing the points of C (to each other, and into and out of P_k). However, this simple strategy does not work, because in P_k, the beacons B_{k+1} may not be a routing set for the region P_{k+1}. This happens because the points of V have changed status from boundary to non-boundary.

We will call the rectangle of C that contains the vertical V the *detachment rectangle*, and the rectangle of P_{k+1} containing V the *base rectangle*. By considering whether the detachment rectangle is a tall, solo, or paired neighbor of the base (analysis omitted in this paper), we find the only problematic case is when it is paired.

In this case, the beacon routing of P_{k+1} may have segments dependent on V being boundary: a routing path segment may encounter the wall of P_{k+1} at a point on V, and then be pulled along that wall containing V until it reaches the reflex vertex (and then leaves the wall; see Figure 5a). In P_k, the corresponding attraction path, upon encountering V, would continue into C and become *trapped*, not reaching the beacon, as shown in Figure 5b.

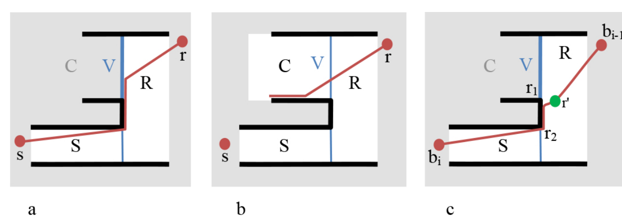

Figure 5: A trapped path. (a) a path segment from r to s hits a wall in P_{k+1}. (b) the attraction path from r towards s continues into C in P_k. (c) repairing a segment from b_i to b_{i-1} with r'.

3.3 Repair of trapped paths

To fix the problem of trapped paths, we will have to devote a new beacon to *repair* such trapped path segments, as suggested in Figure 5c.

Lemma 3 *Let B_{k+1} be a local routing set of beacons in P_{k+1}. If a left (or right) paired neighbor Q has been cut from rectangle R in P_k to form P_{k+1}, we can add the point $r + \varepsilon\hat{x}$ (or $r - \varepsilon\hat{x}$) to B_{k+1} to obtain a beacon set* *that supports local routing between any pair of points in the subpolygon P_{k+1} of P_k, where r is the reflex vertex of P_k common to Q and R.*

The proof of this lemma, lengthy and omitted here, relies crucially on the locality of path segments. If a segment is trapped, then locality allows us to contain that segment in the union of S and R (as in Figure 5), and one other rectangle. The other rectangle must be some sort of neighbor of S or R, and we treat each such possibility in a case analysis.

We use the term *repair position* to refer to the placement of the new beacon (point) in the previous lemma.

4 Upper bound

We will prove the theorem by induction on the size of the dual tree of the vertical decomposition. We first root the dual tree at an arbitrary leaf. At each step, we will examine the structure of the vertical decomposition at and around a deepest node in the rooted tree. We will place some beacons and remove some rectangles/dual tree nodes; we will place at most two beacons per three rectangles removed. We stop and consider basis cases when the depth of the dual tree reaches 0, 1, or 2.

We start with a tree T_0 that is the entire dual tree of the polygon P (which we also denote by P_0). After step k, we will have a tree T_k which is a subgraph of T_0, with the rectangles corresponding to its vertices forming a single polygon P_k which is a subpolygon of P. We call each induction step from T_k and P_k to T_{k+1} and P_{k+1} a *reduction*.

Since the case analysis that will follow gets tedious, we first establish easily-verified sufficient (but not necessary) conditions to form an beacon routing set by inductively cutting off a region C from P_k to yield P_{k+1}. We use these conditions for most but not all of our cases.

Lemma 4 *If the following conditions hold, then $B_k = B_{k+1} \cup B'$ is a routing beacon set for P_k.*

1. *The beacons given (B') cover the region $C = P_k \setminus P_{k+1}$, using local paths.*
2. *B' induces a strongly connected graph in the graph of the local attraction relation.*
3. *At least one element b' of B' is in P_{k+1}.*
4. *If the base rectangle is taller than the detachment rectangle, then b' is in repair position.*

Assume we are after step k, having tree T_k and polygon P_k remaining. If T_k is of height 1 or 2, we stop. Otherwise, let L be a deepest node in the dual tree, let A_1 be its direct ancestor (parent), and in general let A_j be the direct ancestor of A_{j-1}. The grandparent A_2 of L exists. In general, we will try to reduce the size of T_k by removing the dual tree nodes of A_1's subtree, cutting the polygon between A_1 and A_2. In some cases, we must consider alternatives to this cutting location.

The figures used in the case analysis obey the following visual conventions: Parts of the figure boundary *known* to be boundary of P_k are shown with thick black lines. Parts without may or may not be boundary of P_k. Beacon placements are shown as green dots, and rectangles removed in the reduction are shaded.

We assume without loss of generality (by symmetry) that A_2 is an upper right neighbor of A_1. With respect to A_1, the neighbor A_2 is either tall, solo, or paired. We first examine the case when A_2 is taller than A_1.

In this paper, we will only outline this case, giving just one proof; we will furthermore completely omit the three sections for the other cases.

4.1 Case 1: A_2 is a tall neighbor of A_1

In this case, A_1 must have at least one child (the deepest leaf L) and can have at most two children. All of A_1's children are left children.

Lemma 5 *If A_2 is a tall upper right neighbor of A_1, and A_1 has two children, then P_k can be reduced by 3 rectangles at a cost of 2 beacons.*

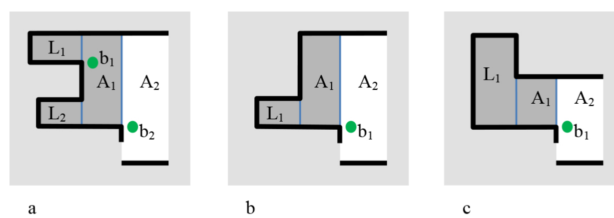

Figure 6: A_2 is a tall neighbor of A_1. (a) A_1 has two children L_1 and L_2. (b) A_1 has a solo lower-left child. (c) A_1 has a tall lower-left child.

Proof. The two children L_1 and L_2 must be left paired children, as shown in Figure 6a.

In this situation, we remove 3 rectangles (L_1, L_2, and A_1) at a cost of placing 2 beacons (b_1 and b_2). Now we show that, if P_{k+1} has a set B_{k+1} of beacons that allows a routing, then P_k has a set of beacons $B_k = B_{k+1} \cup \{b_1, b_2\}$ that allows a routing.

Let $C = P_k \setminus P_{k+1}$, i.e. C is the union of the rectangles L_1, L_2, and A_1. Also let $B = \{b_1, b_2\}$. Now the conditions of Lemma 4 are seen to be satisfied: b_1 covers the cut-off rectangles L_1, L_2, and A_1 (by Lemma 2); b_1 and b_2 are visible, so B' is strongly connected in the attraction graph, and b_2 is in repair position in P_{k+1}. ∎

Lemma 6 *If A_2 is a tall upper right neighbor of A_1, and A_1 has one lower-left child, then P_k can be reduced by 2 rectangles at a cost of 1 beacon (see Figure 6b and 6c).*

Lemma 7 *If A_2 is a tall upper right neighbor of A_1, and A_1 has one short upper-left child, then P_k can be reduced by 2 rectangles at a cost of 1 beacon (see Figure 7a).*

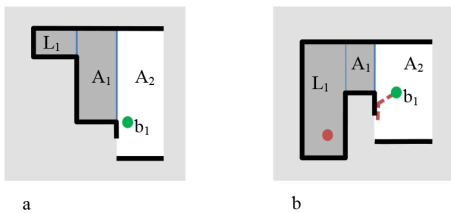

Figure 7: A_2 is a tall neighbor of A_1. (a) A_1 has a short upper-left child L_1. (b) A_1 has a tall upper-left child L_1; the point b_1 is not attracted by the point in L_1.

Figure 7b shows the situation when L_1 is a tall upper-left child of A_1. Here a beacon at b_1 would not suffice, as any point of L_1 below b_1 would not attract b_1. The technique we use to handle this case involves analyzing A_2 and all of its descendants.

4.2 The induction basis

The basis cases are when there are only one to three levels in the dual tree. If it is one level, the polygon is a rectangle. If it is two levels, the polygon is a 6-vertex "L" shape. In both of these cases, every point in the polygon attracts every other point in the polygon (see Lemma 1). Thus, there are no intermediate beacons required and the smallest beacon routing set is of size 0. We omit the analysis for a three-level dual tree.

4.3 The result

Theorem 8 *Any orthogonal polygon of n vertices has a local beacon routing set of at most $\lfloor \frac{n-4}{3} \rfloor$ beacons.*

Proof. Let r be the number of rectangles in the vertical decomposition of the polygon. Since $n = 2r+2$, the floor in the theorem is equivalent to $\left\lfloor \frac{(2r+2)-4}{3} \right\rfloor = \lfloor \frac{2r-2}{3} \rfloor$. We proceed to prove that there is a beacon set no larger than this, by induction on r.

Our basis has $r = 1$ to 6, with each case having a local beacon routing set of 0, 1, or 2 beacons, as discussed above. The number of beacons in each of the cases satisfies $b \leq \lfloor \frac{2r-2}{3} \rfloor$.

For our inductive step, assume $r \geq 3$ and we have rooted the dual tree at a leaf, so the depth of the dual tree is at least 2. One of the lemmas from the case analysis will apply, giving a reduction of 2 rectangles for 1 beacon, 3 rectangles for 2 beacons, 4 rectangles for 2 beacons, or 5 rectangles for 3 beacons. In each of these cases, we show that the local beacon routing set has at most $\lfloor \frac{n-4}{3} \rfloor$ beacons.

Take the first case: here we reduce P by 2 rectangles to construct a P' with $r' = r - 2$ rectangles. By induction P' has a local beacon routing set of at most $\left\lfloor \frac{2r'-2}{3} \right\rfloor = \left\lfloor \frac{2(r-2)-2}{3} \right\rfloor = \lfloor \frac{2r-6}{3} \rfloor$ beacons. To construct the beacon set for P, we add 1 beacon to that, and so we have at most $\lfloor \frac{2r-6}{3} \rfloor + 1 = \lfloor \frac{2r-3}{3} \rfloor \leq \lfloor \frac{2r-2}{3} \rfloor$ beacons.

The other cases proceed in an identical fashion, and the theorem follows. □

5 Lower bound

Here we establish that, for infinitely many n, there are orthogonal polygons that require at least $\left\lfloor \frac{(n-4)}{3} \right\rfloor$ beacons in any routing set. The examples are geometrically simple: each is an orthogonal spiral polygon with a "corridor width" of 1. Bae, Shin, and Vigneron have independently developed similar orthogonal lower-bound examples for the beacon-based art gallery problem [1].

Our polygons will spiral outwards clockwise as one moves through the reflex chain when walking counterclockwise around the polygon (i.e. left hand on interior). Call the reflex vertices of the polygon $r_1, r_2, \ldots r_{(n-2)/2}$ in this counterclockwise order, and let r_0 and $r_{n/2}$ denote the convex vertices adjacent to r_1 and $r_{(n-2)/2}$, respectively. Let c_k be the convex vertex just outside of (and closest to) r_k (refer to Figure 8). Let e_k be the edge from r_k to $r_k + 1$, and l_k be the length of e_k.

Now let C_k be the "corner" 1 by 1 square in P with vertices r_k and c_k, and H_k be the "hallway" rectangle (with dimensions 1 by l_k) between C_{k-1} and C_k.

If m_k^{in} is the midpoint of r_{k-1} and r_k, and m_k^{out} is the midpoint of c_{k-1} and c_k, we can partition the "hallway" H_k into two halves H_k^+ and H_k^- by splitting with its bisector $m_k^{\text{in}} m_k^{\text{out}}$. Let H_k^+ be the half adjoining C_k, and let that half (and not H_k^-) contain the points on the segment $m_k^{\text{in}} m_k^{\text{out}}$.

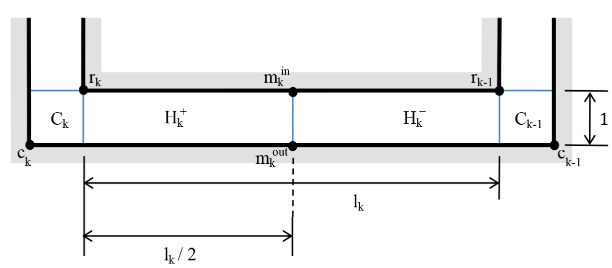

Figure 8: Notation for an orthogonal spiral.

We will construct polygons for $n = 6r + 4$ for some r; these polygons are specified simply by giving the lengths $l_1, l_2, \ldots l_{3r+1}$ of the $3r + 1$ "hallway" rectangles. Provided we have $l_j > l_{j-2} + 2$ for all $3 \leq j \leq 3r$, the polygon will spiral outward and not self-intersect.

We specify r sections $S_1, S_2, \ldots S_r$ of the polygon, by letting S_i be the union of $H^+_{3i-2}, C_{3i-2}, H_{3i-1}, C_{3i-1}, H_{3i}, C_{3i}$, and H^-_{3i+1} (see Figure 9). Note that no point of P is contained in more than one section, and there are points at either end of the spiral (in H^-_1 and H^+_{3r+1}) that are in no section.

Now consider a set of beacons B that can route in such a polygon P. We claim that $|B| \geq 2r$. If this were not the case, then by the pigeonhole principle some section S_i would contain fewer than two beacons.

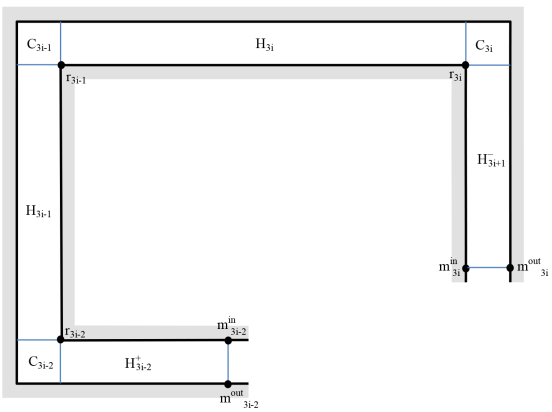

Figure 9: A section of an orthogonal spiral.

In the full paper, we proceed to show that if S_i contains only one beacon, then this beacon must lie in C_{3i-1}. In order to route from points "before" the section to points "after" it, and vice-versa, the beacon must lie in the shaded region in Figure 10, above the line $r_{3i-1}m^{\text{out}}_{3i+1}$ and below the line $r_{3i-1}m^{\text{out}}_{3i-2}$ (directions relative to the figure). By making l_{3i} (the vertical corridor on the right) long enough, we can cross these lines, leaving the reflex vertex r_{3i-1} as the only possibility for the beacon location.

Showing that this reflex vertex cannot properly be attracted to points both before and after section S_i is a purely *definitional* problem. A robot on a reflex vertex is a peculiar thing. There are many possible ways to define what happens when one pulls it towards the exterior: the robot path is indeterminate, the robot follows the wall to the left when it faces the direction of pull, the robot follows the horizontal wall, etc. The model of attraction must address this question somehow.

However, for reasonable, simple models, including those above, such a point r_{3i-1} cannot be successfully attracted both to points before and points after S_i. Thus, in these models, we have a contradiction;

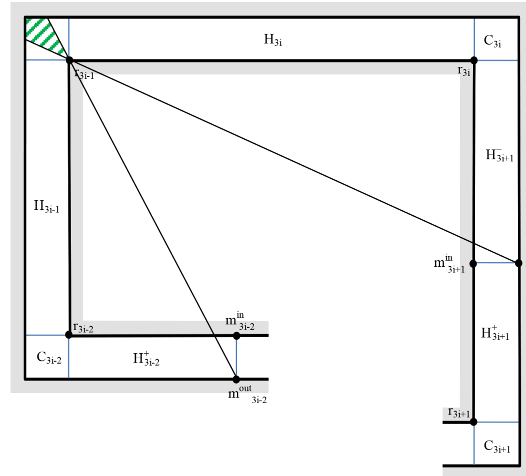

Figure 10: The beacon in S_i must lie in the shaded area.

each section must contain at least two beacons. Hence $|B| \geq 2r$. Since $n = 6r + 4$, $2r = \frac{n-4}{3}$, and we have:

Theorem 9 *For all $n \equiv 4 \pmod 6$, there are orthogonal spiral polygons requiring at least $\frac{n-4}{3}$ beacons in a routing beacon set.*

The constraint on the length of the spiral corridors in Section 5 works out to:

$$l_{3i+1} > \frac{4l_{3i}(l_{3i-1} + 1)}{l_{3i-2}},$$

whose solution is $l_k \in 2^{\Theta(k^2)}$. This growth rate is quite high, leaving us unable to provide figures illustrating these polygons. It would be interesting to try to develop alternative examples that do not have exponentially-growing coordinates.

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