Oblivious Plans for Orienting and Distinguishing Polygonal Parts

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In manufacturing processes, a stream of like parts must often be oriented before assembly. A parts feeder is a mechanism that orients parts: conceptually, a stream of parts in arbitrary orientations is fed into the device from one end, and from the other end the parts emerge in a single fixed orientation. Natarajan [Nat89] formalized an abstract notion of plans for orienting parts, as a sequence of operations on parts designed to bring them into desired orientations, and studied the complexity for certain classes of operations. Goldberg [Gol], Goldberg, Mason and Erdmann [MW85, GME91] and Rao and Goldberg [RG91] considered the problem of generating such plans for planar polygonal parts under simple pushing and grasping operations performed by a common parallel–jaw gripper, without sensors. Each operation reorients the part, but reveals no information about either the initial or final orientation of the part. (Hence the plan is oblivious, inasmuch as it needs no information on the orientation of the part to affect the control structure of the algorithm to achieve correct results.) Using these operations, they were able to construct an algorithm to generate a plans for orienting any planar polygonal part (up to symmetry) in $O(n^2)$ steps. Because of experimental evidence, they also conjectured that only $O(n)$ steps were required to orient any polygon in this model, which was shown in [Gol] to imply the existence of an $O(n^2)$ bound on the time for constructing such plans for any polygon.

In this paper we prove Goldberg's conjecture, and establish tight upper and lower bounds on the number of steps necessary to orient a planar polygonal part in this and in some related models. We also extend these results to the following related problem. A gripper, equipt with a single sensor for detecting the distance between its parallel jaws upon closure, is capable of making ***diameter probes*** of a given polygon. We give a linear upper bound on the number of such diameter probes needed to distinguish among a finite collection of known polygons.

Because of space limitations, we consider only a single model, and omit certain details in constructions and proofs.

1 Polygonal Parts

Natarajan [Nat89] formalized this planning problem in terms of a set of functions — called *transfer functions* — mapping orientations to orientations for some fixed part, modeling the action of some physical device for effecting the desired reorientation. The objective is to construct an oblivious plan for that part, conceived as a sequence of such functions $f_1, \ldots, f_k$ such that

$$f_k \circ f_{k-1} \circ \cdots \circ f_1(S^1) = \{g\}$$

for a fixed goal orientation $g \in S^1$. Natarajan also identified a subclass of transfer functions for which plans and their construction are provably simpler. As in [Gol], we consider only planar polygonal parts. A part is ***polygonal*** if its convex hull is a polygon.

1.1 Transfer functions

Let P be a polygonal part. We identify the set of orientations of a polygonal part in the plane with points in S^1, the unit circle. Following Natarajan, we say that a sequence of n distinct orientations $s_1, \ldots, s_n$ is called ***ordered*** if there is an appropriate identification of S_1 with the real interval $[0, 2\pi)$ under which $s_i < s_{i+1}$ $(i = 1, \ldots, n-1)$. A function $f : S^1 \to S^1$ is called ***monotonic*** (or order-preserving) if for every ordered sequence of orientations $s_1, \ldots, s_n$, the sequence $f(s_1), \ldots, f(s_n)$ is also ordered.

By a ***transfer function***, we mean a function $f : S^1 \to S^1$ which models some feasible primitive operation on orientations. Goldberg et al. showed that a useful set of transfer functions may be effectively constructed for polygonal parts under the action of a parallel–jaw gripper [MW85, GME91]. In this abstract, we consider the following types of actions. (A plan will use only one type.) In a ***pure squeeze*** action, the gripper in a given orientation closes and grasps the part, causing the part to rotate in the plane until it reaches a stable configuration (where

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the gripper can close no further). In this case, the stable orientations correspond to local minima of the polygon's diameter function — the mapping from orientations to diameters of the polygon. Although conceptually and computationally simpler, pure squeezing unrealizeable in practice, since some "pushing" is unavoidable, ***Push–grasp*** actions were proposed as a more realisitic alternative. In this case, the gripper is put into a given orientation and one surface of the gripper is swept across the plane, causing the polygon to rotate into a stable contact with the sliding surface. At this point the second jaw of the gripper is closed on the polygon, perhaps causing it to rotate into another stable orientation. The transfer function realized by the two phases of the action — pushing and grasping — can be computed from a knowledge of the geometry of the part, its centroid, and its diameter function. Composing these yields the transfer function induced by the operation on a given polygon. The construction is discussed in [Gol]. In both of these cases, the relation between initial and final orientations is not necessarily functional, although it is many–one on all but a finite number of orientations. For convenience in this abstract, we assume that the transfer function is extended to a total function consistent with this relation.

For either action, there is a family of transfer functions, where each member of the family is generated by placing the gripper in a different orientation (relative to the world coordinates). For a fixed polygon, any two such transfer functions are related by a simple translation of S_1. So if f_{α_i} is the function induced by the gripper in orientation $\alpha_i \in S_1$ $(i = 1, 2)$, then if $f_{\alpha_1}(s) = t$ it follows that $f_{\alpha_2}(s - \alpha_1 + \alpha_2) = t - \alpha_1 + \alpha_2$. We generally shall fix one distinguished orientation (say $0 \in S^1$) for the gripper and call the induced transfer function f *the* transfer function of the polygon. When α is the actual orientation of the gripper in the world coordinates, then the part, when in orientation $a + \alpha$, is mapped to $f(a) + \alpha$.

The relation induced by either pure–squeezing and push–grasps on a fixed polygonal part has the following properties.

1. It is functional on all but at most a finite number of points. We let M be the finite set of points for which this relation is not many–one.

2. f is monotonic (order–preserving).

3. Write $F = f(S^1 \setminus M)$ for the set of fixed points of f. Then F is finite. The sets $f^{-1}(a)$, as a ranges over points in F, comprise the connected components of $S^1 \setminus M$.

For convenience, we shall assume that for all $a \in F$, $f^{-1}(a) = (b, c)$ for some $b, c \in M$. (This merely maximizes the cardinality of M.) In this case, the transfer function f is uniquely determined by F and M, and we write $f = (F, M)$.

The complexity of the plans constructed in the next section will be measured in terms of the complexity of f, where $n = |F| = |M|$. It is easy to see that for the above actions, n is never more than three times the number of edges in the convex hull of a given part. Below we use the term "transfer function" to refer to functions satisfying these criteria. For completeness, we assume only that when $m \in M$, $f(m) \in \{m, f(m \pm \epsilon)\}$ for every sufficiently small ϵ. This models the situations which arise above, where a point in M may be an isolated stable orientation, or may be mapped into the "nearest" stable orientation in F.

We also consider only aperiodic transfer functions. It is easy to see that a function with a periodic transfer function can, at best, be oriented up to symmetries (of the transfer function) by these actions.

Finally, we assume that all orientations of the polygon are expressed relative to the orientation of the gripper. Suppose that we start with the gripper in orientation β and execute the following steps: (1) reorient the gripper by α; (2) apply the gripper. Then $a \stackrel{\alpha}{\Longrightarrow} b$ relates orientations a of the polygon, relative to the original orientation β of the gripper, and the final orientation b of the polygon, under the induced action, now expressed relative the new orientation $\beta + \alpha$ of the gripper. Similarly, we write f_α for the function $f_\alpha(a) = f(a - \alpha)$.

1.2 Additional notation

Let $f = (F, M)$ be a monotone, aperiodic transfer function with $|F| = n$. We shall generally assume a fixed ordering has been chosen for elements of $F \cup M$. Since $f^{-1}(a)$ is an open subset of S^1, we may also define $\{b \in f^{-1}(a) : b < a\}$ in the natural way. For any fixed point $a \in F$, we define

$$\begin{aligned} l(a) &= \{b < a : f(b) = a\} \\ r(a) &= \{b > a : f(b) = a\} . \end{aligned}$$

2 Constructing $O(n)$-Step Plans

In this section we prove that for every monotone, aperiodic transfer function $f = (F, M)$ and every goal orientation g, there is an oblivious plan of fewer than $2|F|$ steps which orients any polygonal part with this transfer function. By a ***fixed, oblivious plan of m-steps***, we mean a sequence of reorientations for the gripper $(\alpha_1, \ldots, \alpha_m)$ relative to some fixed initial orientation in the world coordinate system. Without loss of generality we assume that the initial orientation of the gripper is 0. We say that an orientation $a \in S^1$ is an ***active orientation at step i*** if

$$s_0 \stackrel{\alpha_1}{\Longrightarrow} s_1 \stackrel{\alpha_2}{\Longrightarrow} s_2 \stackrel{\alpha_3}{\Longrightarrow} \cdots \stackrel{\alpha_{i-1}}{\Longrightarrow} s_{i-1} \stackrel{\alpha_i}{\Longrightarrow} s_i = a$$

for some $s_0, \ldots, s_{i-1} \in S^1$. The plan correctly orients the polygon to the goal orientation g if $g - \sum_{i=1}^{m} \alpha_i$ is the only active orientation at step m. We observe that if there is a plan which brings a polygon into the orientation g, then we can always recover a plan for the goal g'.

Lemma 2.1 *If $(\alpha_1, \ldots, \alpha_m)$ is a plan which orients a part to orientation g, then for any orientation g' $(\alpha_1 + [g' - g], \ldots, \alpha_m)$ is a plan which leaves the polygon in orientation g'.*

Hence it suffices to construct plans which bring every initial orientation into an arbitrary but fixed goal orientation.

2.1 A Special case

Under our assumptions, the two steps suffice to reduce the set of all active orientations to those in F: the first step reduces the problem to orientations in $F \cup M$, and one more appropriate grasp will reduce these to a subset of F. This immediately implies that the active orientations at *every* step $i \geq 2$ form a subset of F. Let $a_0, \ldots, a_{n-1}$ be an ordered enumeration of the fixed points in F, with $a_i < a_{i+1 \bmod n}$ for all $i = 0, \ldots, n-1$. Let $c = \max_{a \in F} |r(a)|$. We first consider those transfer functions for which there is a *unique* fixed point — say a_0 — such that $|r(a_0)| = c$. For any such f we have the following plan. Let $\epsilon > 0$ be a small constant such that (1) $c - \epsilon > |r(a_j)|$, and (2) $a_j - c + \epsilon \notin M$ for all $j \neq 0$. It is clear that such an ϵ always exists.

1. Reduce the problem to active orientations in F only.
2. for each $i = n-1, \ldots, 1$,
   (a) reorient the gripper by $c - \epsilon$, moving a_i to $a_i - c + \epsilon$; and
   (b) apply the gripper.

We show that for any initial orientation s of the polygon, this plan puts the polygon into the fixed orientation a_0 (relative to the final orientation of the gripper).

Proof. By our assumptions on the transfer function f, after the first application in Line 1, the only active orientations are fixed points in the set F. Each successive application of $f_{c-\epsilon}$ moves points in F to points in F. Continuing through the plan, at each step we reorient the gripper by $c-\epsilon$ and apply it through a single application of $f_{c-\epsilon}$. For each fixed point a_{i_1} this yields a sequence of fixed points $a_{i_1} \stackrel{c-\epsilon}{\Longrightarrow} a_{i_2} \stackrel{c-\epsilon}{\Longrightarrow} \cdots \stackrel{c-\epsilon}{\Longrightarrow} a_{i_n}$ with each $a_{i_j} \in F$. It suffices to prove that for any $a_{i_1} \in F$, $a_{i_n} = a_0$. So let a_j be any fixed point, $j > 0$. Since $|r(a_j)| < c - \epsilon$, by our choice of c and ϵ, $a_j - c + \epsilon \notin f^{-1}(a_j)$. Since $\stackrel{c-\epsilon}{\Longrightarrow}$ maps F into F, it follows that $f_{c-\epsilon}(a_j - c + \epsilon) = a_k$ for some $a_k \in F$. It is easy to see that $a_k \in \{a_0, \ldots, a_{j-1}\}$, by the maximality of c and the fact that f is order-preserving. This proves that the sequence of indices $i_1, i_2, \ldots$ is strictly decreasing until we reach some $i_j = 0$.

Since $c = |r(a_0)|$ and $c > \epsilon > 0$, $f_{c-\epsilon}(a_0) = a_0$. So once a fixed point has been collapsed to a_0, it is mapped to a_0 by each successive iteration of the loop at Line 2. The correctness of the plan follows immediately. Clearly, no more than $n+1$ steps are required. □

2.2 The General case

As above, let c maximize $|r(a)|$ as a ranges over points in F, and let $a_0, \ldots, a_{n-1}$ be an ordered enumeration of F. Note that $c \geq \pi/n$, and $|f^{-1}(a)| \leq 2c$ for all $a \in F$.

The algorithm of the previous section fails when there is no *unique* a_i for which $r(a_i) = c$. Let $a_{i_0}, \ldots, a_{i_{k-1}} \in F$ be an ordered enumeration of the fixed points for which $|r(a_{i_j})|$ realizes this maximum. (These are just the fixed points of $f_{c-\epsilon}^{n-1}$.) To adapt these plans to the general, case we need the following lemma.

Lemma 2.2 (Stretching Lemma) *Let $0 < d < 2\pi$. Then if f does not have period d, there are orientations $a, b \in S^1$ such that $d = |(a,b)|$ and $d < |(f(a), f(b))|$.*

Proof. Partition S^1 as follows. Let $L = \bigcup_{a \in F} l(a) \setminus M$. Since F and M are disjoint, this is necessarily a non-empty, open set. The orientations in L are candidate locations for the point a, the points for which $f(a) < a$. Since b must be located at a distance of d from a, this constrains b to lie in $L + d$. Let $R = \bigcup_{a \in F} r(a) \setminus M$. Now if we can put a in L, and b in R, then $f(a) < a$ and $f(b) > b$, and so the image of the interval (a, b) under f has grown. To achieve this, we must have $b \in R \cap (L + d)$. So if this set is non-empty, there exists such a choice.

We next show that $R \cap (L+d)$ is non-empty whenever f does not have period d. For a contradiction, assume that $R \cap (L+d)$ is empty. Then R is contained in $S^1 \setminus (L+d) = (R+d) \cup F \cup M$. But $R + d \cup F \cup M$ is just the closure of the open set $R+d$. This implies that R and $R+d$ are compact open sets of the same measure. But if $R \subseteq R+d$, then $R = R+d$, from which it follows that f has period d. So $R \cap (L+d)$ is non-empty whenever f is does not have period d. It follows that if we choose $b \in R \cap (L+d)$, and set $a = b - d$, then the statement of the lemma is satisfied. □

As a consequence, it easily follows that for any $a, b \in F$, with $a < b$, there are always orientations α, β such that the following steps map b to $b + \delta$ (for some δ, $0 < \delta \leq 2c$) while leaving a fixed: (1) reorient the gripper by α; (2) apply the gripper; and (3) reorient the gripper by β. This is a straightforward application of the Stretching Lemma. Note also that, if there are $m \geq 0$ active orientations before this sequence of actions, then there are no more than m active orientations afterwards.

As above, let $a_{i_0}, \ldots, a_{i_{k-1}}$ be an ordered enumeration of those fixed points for which $|r(a_{i_j})| = c$. Choose ϵ such that (1) $c - \epsilon > |r(a_m)|$, (2) $a_m - c + \epsilon \notin M$ and (3) $\epsilon < \delta_j$, for all $m = 1, \ldots, n-1$ and $j = 0, \ldots, k-1$. We then consider the following plan.

1. Reduce to active orientations in F only.
2. For each $j = k-1, \ldots, 1$ do:
   (a) While there are active orientations in (a_{i_j}, a_{i_0}): reorient the gripper by $c - \epsilon$, moving a_{i_j} to $a_{i_j} - c + \epsilon$, and apply the gripper.
   (b) If $j = 0$ then
      i. move a_{i_j} to $a_{i_j} - \delta_j$ while leaving a_{i_0} fixed (for some $0 < \delta_j \leq 2c$), using the Stretching Lemma;
      ii. reorient the gripper by $c - \epsilon$, moving $a_{i_{j-1}}$ to $a_{i_{j-1}} - c + \epsilon$; and
      iii. apply the gripper.

Proof. The arguement of the previous section shows that, for each j, the loop in Line 2a collapses all fixed points between a_{i_j} and $a_{i_{j+1}}$ into a_{i_j} in a number of steps bounded by the number of points in $F \cap (a_{i_j}, a_{i_{j+1}})$. After the application of the Stretching Lemma in Line 2(b)i to the points a_{i_j} and a_{i_0}, a_{i_j} is moved to $a_{i_j} - \delta_j$ (some $\delta_j > 0$). This moves the image of a_{i_j} closer to a_{i_0}, so that $f(a_{i_j} - c - \epsilon - \delta_j) = a_m$ for some $m < i_j$. Hence the action at Line 2b will always map the fixed point a_{i_j} *into a_k for some $k < i_j$. We can then show that, for any* $a \in F$, $a \neq a_{i_0}$, whenever there are no active orientations in (a, a_{i_0})

1. if $a \notin \{a_{i_0}, \ldots, a_{i_{k-1}}\}$, then each iteration of the loop at Line 2a there are no active orientations in (a, a_{i_0}), and
2. if $a = a_{i_j}$ ($j = 1, \ldots, k-1$), then after executing Line 2b there will be no active orientations in (a_{i_j-1}, a_{i_0}).

So when the plan terminates, the only active orientation is a_{i_0}. Since grasp leaves the point a_{i_0} fixed, this is the only active orientation upon termination.

For the complexity: At most two step reduce S^1 to n active orientations. For each of the fixed points $a_1, \ldots, a_{k-1}$ we use two steps in Line 2a. The loop at Line 2b is iterated at most once for each fixed point not among $a_{i_0}, \ldots, a_{i_{k-1}}$. So the plan requires $n + k$ steps. □

This proves the following theorem.

Theorem 2.3 *Let $f = (F, M)$ be a monotonic, aperiodic transfer function with $n = |F|$. Let*

$$c_l = |\{a \in F : l(a) = max_{a \in F}|l(a)|\}|$$
$$c_r = |\{a \in F : r(a) = max_{a \in F}|r(a)|\}|$$

and let $k = max(c_l, c_r)$. Then there is a plan of $n+k < 2n$ steps for orienting any polygon with transfer function f.

Given a description of a polygon, it is straightforward to show that such a plan can be constructed in $O(n^2)$ time.

3 Lower Bound

We show that for every n and k ($0 \leq k < n$) there is a transfer function $f_{n,k}$ (with parameters n, k) for which the shortest plan has $n+k-1$ steps. Here we consider only the case $k = n-1$; the construction is easily generalized. By the results of Rao *et al.* [RG91], every transfer function $f = (F, M)$ is the transfer function of a polygon with at most $2n$ edges under pure squeezing.

Let $f = (F, M)$ where

$$M = \{2, 4, 6, \ldots, 2n-4, 2n-2\}$$
$$F = (M - \frac{1}{2}).$$

f defines a transfer function on $\mathbf{R} \bmod (2n-1)$. To simplify the argument in this abstract, we assume that f is undefined on points in M.

Now suppose that there is a plan for orienting a polygon with transfer function f to goal orientation g given by the sequence of orientations $a_1, \ldots, a_m$. On the ith step, we apply the function f_{a_i} to all active orientations. We work backwards. Let J be any connected open subset of $\mathbf{R} \bmod (2n-1)$ of integral size $j = 2, \ldots, 2n-1$.

Claim 3.1 *For any s, $|f_s^{-1}(J)|$ is an open interval of size at most $j+1$.*

Proof. If $j = 2j'+1$, then the largest preimage is obtained when J covers j' fixed points a, each of which satisfies $|f^{-1}(a)| = 2$. Hence, for an appropriate s, $|f_s^{-1}(J)| = 2j'+1 = j+1$, and for all choices of s, $|f_s^{-1}(J)| \leq j+1$. If $j = 2j'$, the interval J can be made to cover j' fixed points a for which $|f^{-1}(a)| = 2$, together with the unique fixed point b for which $|f^{-1}(b)| = 1$. Hence for an appropriate s, $|f_s^{-1}(J)| = 2j'+1 = j+1$, and for all other choices $|f_s^{-1}(J)| \leq j+1$. The openness follows from the assumptions on the behavior of f at points in M. □

Now any correct plan $f_{a_1}, \ldots, f_{a_m}$ must satisfy $f_{a_m} \circ \cdots \circ f_{a_1}([0, 2n-1)) = \{g\}$ for some g. By the previous observations, $|f_{a_m}^{-1}(g)| \leq 2$ and, from the claim above and a simple induction, $|(f_{a_i} \circ \cdots \circ f_{a_m})^{-1}(g)| \leq m - i + 1$. So $m = 2n-3$ steps are required to get a preimage of size $2n-1$; and since this is still a proper open subset of $[0, 2n-1)$, an additional step is needed to include all of this interval.

4 Composing Plans

Suppose that we have m parts $P_1, \ldots, P_m$ with transfer functions $f_1, \ldots, f_m$ respectively. Then for each i, we can construct a plan of no more than $2|f_i| - 1$ steps which brings part P_i into a known orientation g_i. In addition, it is straightforward to show that by composing these plans — performing them in sequence, one after the other — we obtain a general plan of $2(\sum_{i=1}^{m}|f_i|) - m$ steps which will bring any of these m parts into a known orientation. The following lemma follows immediately from the construction presented above.

Lemma 4.1 *Let $\pi = (\alpha_1, \ldots, \alpha_k)$ be a plan for orienting a polygon with transfer function f into the orientation g. Then there is an open neighborhood U_i of each α_i such that every plan $\pi' \in U_1 \times \cdots \times U_k$, will take the given polygon into the orientation g.*

4.1 Composition of plans

Suppose that we have m parts $P_1, \ldots, P_m$ with transfer functions $f_1, \ldots, f_m$. Then for each i, we can construct a plan of no more than $2|f_i| - 1$ steps which brings part P_i into a known orientation g_i. In addition, it is straightforward to show that by composing these plans — performing them in sequence, one after the other — we obtain a general plan of $2(\sum_{i=1}^{m}|f_i|) - m$ steps which will bring any of these m parts into a known orientation.

Lemma 4.2 *There are orientations $\gamma_1, \ldots, \gamma_m$, and a single plan of at most $2(\sum_{i=1}^{m}|f_i|) - m$ steps which, when applied to any polygon $P_i \in \{P_1, \ldots, P_m\}$, puts P_i into the fixed orientation γ_i.*

The lemma is a consequence of Theorem 2.3 and the following observation. Write S_m for the disjoint union of m copies of S^1, a representation of the configuration space of a polygon selected from $\{P_1, \ldots, P_m\}$. Write (i, α) for the orientation α in the ith component of S_m. Informally, the element (i, α) corresponds to polygon P_i in orientation α.

By the previous theorem, there exists a plan π_i which puts polygon P_i into a fixed orientation g_i, for each P_i and g_i. For convenience, we write π_i also for the induced function on S_m realized by this plan. By the correctness of the plan, it follows that $\pi_i((i, \alpha)) = (i, g_i)$ for all $i = 1, \ldots, m$ and all $\alpha \in S_m$. On the other hand, it is trivially true that for all j and α, $\pi_i((j, \alpha)) = (j, \alpha')$ for some $\alpha' \in S^1$. If the plans π_i can be constructed to be functional on the given set of polygons — so that $\pi_i((j, \alpha))$ is single valued when $i \neq j$ — it follows that

$$\pi_j((i, g_i)) = (i, g_i')$$

for some fixed orientation $g_i' \in S^1$. A simple induction then shows that the plans $\pi_1, \ldots, \pi_m$, when composed, satisfy

$$\begin{aligned} \pi_m \circ \pi_{i-1} \circ \cdots \circ \pi_1((j, \alpha)) &= (j, \gamma_j) \quad \text{if } j < i \\ \pi_m \circ \pi_{i-1} \circ \cdots \circ \pi_1((i, \alpha)) &= (m, g_m) \end{aligned}$$

for some sequence of orientations $\gamma_1, \ldots, \gamma_{m-1} \in S_1$. Taking $\gamma_m = g_m$ proves the lemma.

It remains to show that the plans π_i can always be constructed so that they are functional on chosen orientations. Less formally, we need to establish that whenever the first $i-1$ plans $\pi_1, \ldots, \pi_{i-1}$, applied in succession, put the first $i-1$ polygons into fixed orientations, then the orientations used by plan π_i keep each of these polygons in a single orientation. That is, the chosen orientations for π_i do not fall at points where any of the transfer functions f_j may have indeterminate values. Hence it suffices to prove the following lemma.

Lemma 4.3 *Suppose that there are orientations $\gamma_1, \ldots, \gamma_{i-1}$ such that*

$$\pi_{i-1} \circ \cdots \circ \pi_1(j, S^1) = (j, \gamma_j)$$

for every $j \leq i-1$. Then the plan π_i can be modified so that

$$\begin{aligned} \pi_i(i, S^1) &= (i, g_i) \quad \text{and} \\ \pi_i(j, \gamma_j) &= (j, \gamma_j') \end{aligned}$$

for unique values $\gamma_1', \ldots, \gamma_{i-1}' \in S^1$.

This follows from the observation that for each transfer function there are a finite number of "bad" values, and that the actual orientations used in individual plans may be perturbed slightly while still maintaining the desired properties of those plans.

Further details are presented in the full paper.

5 Distinguishing Polygons

As a consequence of the results above on composition of plans, we can also show that a gripper, which is equipped with a single sensor for detecting the distance between its parallel jaws upon closure, can distinguish among a finite collection of known polygonal objects under very general conditions. Define a function d which maps orientations to diameters as follows. For any $a \in S^1$, $d(a)$ is the diameter of the polygonal part in orientation $f(a)$. We call this a sensor map for the proposed gripper. We say that two sensor maps d_1, d_2 are equivalent if there is an orientation $a \in S^1$ such that

$$(\forall x \in S^1)\ d_1(x) = d_2(x + a)\ ,$$

and write $d_1 \equiv d_2$ when this is the case.

Using the previous corollary, we can show that whenever two polygonal parts have distinct sensor maps, they can in fact be distinguished by a short plan, in which the number of steps is linear in the number of edges in the two polygons.

Corollary 5.1 *Let P_1 be a polygon with transfer function f_1 and sensor map d_1, and P_2 a polygon with transfer function f_2 and sensor map d_2. Then if $d_1 \not\equiv d_2$, a gripper equipped with sensors (as described above) can distinguish between these two polygonal parts using at most $2(|d_1| + |d_2|) - 1$ grasps.*

On the other hand, $f_1 \equiv f_2$ and $d_1 \equiv d_2$, then the parts can not be distinguished by the proposed class of plans.

From the previous corollary we know that there are orientations g_1, g_2 and a single plan π which will put any instance of polygon P_1 into the orientation g_1, and any instance of P_2 into the orientation g_2. Using the distinctness of the sensor maps, we can find an orientation α in which to squeeze, such that the measured sensor value will distinguish between the two polygons, using one diameter probe (and hence one additional grasp)..

It is straightforward to extend these results to a collection of polygons, giving the following corollary.

Corollary 5.2 *Let $P_1, \ldots, P_m$ be as above. Then there is a non-adaptive plan of at most $2\sum_{i=1}^{m}|P_i| + \frac{1}{2}m(m-1) - m$ grasps to distinguish a given part P from the collection $\{P_1, \ldots, P_m\}$.*

References

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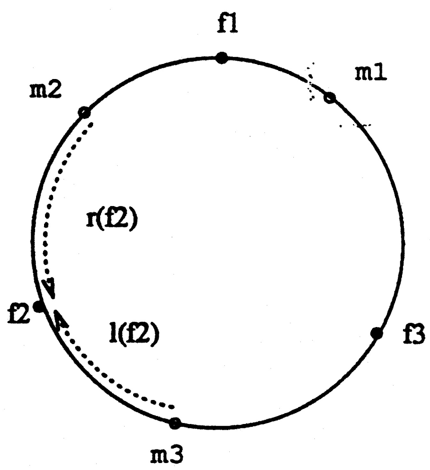

Figure 1: Representation of the function $f = (F, M)$ on S^1, with $F = \{f_1, f_2, f_3\}$ and $M = \{m_1, m_2, m_3\}$. (m_1, m_3) is mapped to f_2 under f, and $l(f_2) \cup r(f_2) \cup \{f_2\} \subseteq f^{-1}(f_2)$.