Geometric Enumeration without Memory

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ABSTRACT

This talk describes a new enumeration technique developped with Komei Fukuda that can be used for a variety of geometric enumeration problems, including:

- enumerating vertices, edges and cells of a hyperplane arrangement
- enumerating vertices and edges of the intersection of a collection of half-spaces.

Using standard transformations, this approach can therefore be used to find the facets of the convex hull of a set of points, the vertices and edges of a Voronoi diagram or the cells in a Delaunay triangulation. The algorithm is based on "reversing" a carefully chosen optimization problem constructed on the given input and has many nice features, including:

- No additional storage is required beyond the input data;
- The output list produced is free of duplicates;
- The algorithm is extremely simple, requires no data structures, and handles all degenerate cases; The running time is output sensitive for non-degenerate inputs;
- The algorithm is easy to efficiently parallelize.

For example, the algorithm finds the v vertices and edges of a polyhedron in R^d defined by a non-degenerate system of n inequalities (or dually, the v facets of the convex hull of n points in R^d, where each facet contains exactly d given points) in time $O(ndv)$ and $O(nd)$ space.