# Weighted 1-Center Problem in a Simple Polygon

S. Kabadi[1], Y.P. Aneja[2] and K.P.K. Nair[1]

## Extended Abstract

The problem we consider is as follows. We are given n demand points $p_1$, $p_2$, ..., $p_n$ inside a simple polygon P, and n positive weights $w_1$, $w_2$, ..., $w_n$ associated with these n points, respectively. For any point x inside this polygon, define

$d(x,p_i)$ = the length of the shortest path from x to $p_i$ with the entire path inside P.

$F(x) = \text{Max}_i \{w_i d(x,p_i)\}$

The problem is to determine an $x^*$ such that $F(x^*) = \text{Max}_{x \in P} \{ F(x) \}$.

This problem is a generalization of the problem considered by Pollack *et al.* [PSR], where the demand points are restricted to the corner points of the simple polygon and all the weights are 1. We extend the results in [PSR] by providing an $O(n\log n)$ for our problem.

Since our algorithm is a direct generalization of the one in [PSR], we first review the related results. In [M1], Megiddo gave a linear time algorithm for finding the smallest circle enclosing a given set of n points in $R^2$ (the unweighted 1-center problem). The ideas in [M1] were extended by Dyer [D] to produce a linear time algorithm for the weighted 1-center problem in $R^2$. In [M2], Megiddo considered another generalization of the 1-center problem where the weights associated with the demand points are 'additive' instead. That is, given $\alpha_i \geq 0$, i = 1, 2,..., n, the objective defined is

$G(x) = \text{Max}_i \{Ed(x,p_i) + \alpha_i\}$, and we want to determine $x^*$ such that

$G(x^*) = \text{Min}_{x \in R^2} \{ G(x) \}$.

Here, $Ed(x,p_i)$ represents the Euclidean distance between x and $p_i$. The above problem can be looked upon as finding the smallest circle enclosing n given circles. Megiddo [M2] gave a linear time algorithm for this problem by extending the results in [M1] and [D]. This algorithm was used as a subroutine in [PSR] to produce an $(n\log n)$ for computing the geodesic center of a simple polygon. The algorithm in [PSR] starts by considering a

[1]Faculty of Administration, University of New Brunswick, Fredericton, N.B. E3B5A3

[2]Faculty of Business, University of Business, University of Windsor, Windsor, Ontario, N9B3P4

triangulation of the simple polygon P. The 'median' diagonal is tested to determine on which side of this diagonal the geodesic center of P lies. Using this binary search, a triangle containing the center is determined. Further a region, containing the geodesic center, in this triangle is determined such that the combinatorial structure of the paths from the center to the corner points of P remains invariant. The provides the $\alpha_i$'s associated with each of the demand points. Finally using these 'additive' weights, the 1-center problem is solved.

Our algorithm proceeds in a manner similar to that of [PSR]. We start with a triangulation of P and the n demand points. By eliminating certain triangles, we reduce the polygon to obtain another simple polygon so that all the demand points are corner points of this polygon. We then search over the triangulation, using a 'binary search' scheme, to determine the triangle that contains the optimal center. The search is carried out in essentially the same way as in [PSR] and requires as a subroutine an algorithm for the following problems:

i. Given any chord of the simple polygon, find an optimal center given that it is constrained to lie on the chord.

ii. Find on which side of the chord the unconstrained optimum lies.

Problem (i) can be solved in linear time using ideas in [M1]. A result similar to the one in [PSR] enables us to determine the side on which the unconstrained optimum lies. Repeated application of (i) and (ii) yields a triangle containing the unconstrained optimum. Having found this triangle, we further partition the triangle into regions such that, in each such region, the distance from any point x in this region to a demand point $p_i$ can be written as:

$$d(x,p_i) = Ed(x,q_i) + d(q_i,p_i)$$

where, $Ed(x,q_i)$ is the Euclidean distance from x to a known vertex $q_i$ of the polygon. Again, using ideas in [M1], we identify the region of this triangle containing the optimum solution.

Finally, to locate the optimal center $x^*$, we need to consider a 'modified' 1-center problem. where with each demand point, we associate a corner point $q_i$, a 'multiplicative' weight $w_i$ and an 'additive' weight $\alpha_i$. Thus we define

$$H(x) = \text{Max}_i \{w_i.Ed(x,q_i) + \alpha_i\}, \text{ and}$$

$$H(x^*) = \text{Min}_x \{ H(x) \}.$$

This problem can be viewed as finding the weighted 1-center for given n circles in the plane. We provide a linear time algorithm for this generalized 1-center problem, again, essentially using the multidimensional search technique developed in [M1]. This algorithm can be extended to any fixed dimension d.

**References**

[D] Dyer, M. E. (1984). On a multidimensional search technique and its application to the Euclidean 1-center problem, *SIAM J. Computing* 15, pp. 725-738.

[M1] Megiddo, N. (1984). Linear Programming in linear time when the dimension is fixed, *J. Assoc. Computing Machinery*, 31, pp. 114-127.

[M2] Megiddo, N. (1989). On the ball spanned by balls, *Discrete Comput. Geom.*, 4, 605-610.

[PSR] Pollack, R., Sharir, M., and Rote G (1989). Computing the geodesic center of a simple polygon. *Discrete Comput. Geom.*, 4, 611-626.