Minimal Obscuring Sets: the Parallel View Case

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Abstract

Given a simple polygon K contained in the interior of a simple polygon P, and a point s at infinity in some direction, a minimal obscuring set is the shortest subset of the boundary of P that can hide K from s.

An $O(n \log n)$ algorithm is presented that computes a minimal obscuring set for s given P and K. The algorithm is shown to be optimal by a proof of lower bound on the problem.

Let S be a proper subset of the two-dimensional Euclidean space E^2. If x and y are two points of E^2 such that the line segment joining x to y intersects S, then S ***obscures*** x from y, and y from x.

Let e be a line segment of E^2. The set S ***obscures*** e from x if every ray r with origin x intersecting e also intersects S.

Let P and K be simple polygons, with K contained in the interior of P, and let s be a point in the exterior of P.

An ***obscuring set*** A of s with respect to K and P is a subset of the boundary of P such that when included in S it obscures K from s. The ***length*** of the obscuring set, A, denoted by $L(A)$ is the sum of the lengths of the segments in A. A *minimal obscuring set* is an obscuring set of minimal length.

A formal definition of the problem considered here follows.

Parallel View Case of the Minimal Obscuring Set Problem

> Let K be a simple polygon contained in the interior of a simple polygon P, let θ be a direction in the plane, and s a point at infinity in the direction θ. Find a minimal obscuring set for s given K and P.

A *corridor*, C, is a proper subset of the plane, delimited by two distinct parallel lines. Let l be a line orthogonal to a corridor C, and let $t = l \cap C$. we say that t *span* C. The *diameter* of a given corridor is given by the length of the line segment spanning it.

A *sub-corridor* of a corridor C is a corridor contained in C. A *subtending edge* of a corridor C is a line segment, whose endpoints lie on the bounding lines of C. Alternatively, we say that C is *subtended* by e. Given a corridor C and two subtending edges e and e', we say that e is *dominant* over C if $L(e) < L(e')$.

Let $VP(s|K)$ be the visibility polygon of s given K. Let D_l and D_r be the leftmost and rightmost windows of $VP(s|K)$ respectively, when moving clockwise along K.

Let l_s be an orthogonal line segment to θ with endpoints on D_l and D_r and on the same side of s with respect to P. Clearly, hiding K from s amounts to hiding K from l_s.

Let $E = P \cap VP(s|K)$. Of all segments of P, these are the candidates to be in the minimal obscuring set. All elements of E will be called *edges* even though some of them are segments of edges of P.

Let V be the set of vertices of E, and T the set of contiguous intervals of l_s induced by the projection of V onto l_s. Each of the intervals of T span a sub-corridor.

Each sub-corridor must be subtended by a dominant segment. Clearly, the union of these segments forms a minimal obscuring Set. This simple characterization yields the algorithm outlined next.

The Parallel View Case Algorithm

(1) Remove all edges of P not in $VP(s|K)$.

(2) Sort T and V from left to right with respect to l_s.

(3) Over each interval t of T, choose the shortest segment subtending the corridor spanned by t.

The algorithm runs in $O(n \log n)$ be maintaining a heap of edges of E. Each edge is inserted once (when its left vertex is visited) and deleted once (when its right vertex is visited).

Let m be the smallest interval of T, e an edge of E, and e_m any segment of e subtending a sub-corridor which diameter $L(m)$. The pair $(e, L(e_m))$ is inserted in the heap, with $L(e_m)$ as a key.

The choice of $L(e_m)$ as a key for insertion in the heap ensures that the edge at the top of the heap always contains the dominant segment for the present sub-corridor even if some edges subtend several sub-corridors.

A transformation from Element-Uniqueness establishes an $\Omega(n \log n)$ lower bound on the problem. It is outlined next.

Let $I = \{a_1, \ldots, a_n\}$ be a set of positive integers. Each integer a_i is mapped into $((a_i, i), (a_i + 1, i))$, a segment of length one, parallel to the x-axis, termed *flat segment.*

Let a and a' be respectively the minimum and maximum integers in I. Let $K = ((a, n+1), (a', n+1))$.

A polygon, P, is constructed by linking consecutive flat segments along the y-axis and joining the last flat segment to the first via a polygonal chain which 'wraps around' K. Thus K is contained in P. The point s is at infinity in the negative y-axis direction.

In order to answer the Element-Uniqueness Problem, it suffices to read the number of flat edges in the minimal cover that is computed by any given algorithm with the constructed instance as input. If the number of flat segments equals n, then all integers are unique, otherwise two or more integers are equal.