Free-form Surface Modeling Using Implicit Patches

Baining Guo*
Cornell University

March 20, 1990

Free-form surfaces are used to represent curved objects in geometric modeling, computer graphics, computer-aided design and robotics. The construction of free-form surface amounts to the solution of the following problem, which we call the free-form surface problem. Given a continuous piecewise linear surface **L** whose vertices are equiped with normals, construct a smooth piecewise polynomial surface that matches the vertices and corresponding normals.

We search for a method to slove the free-form surface problem using quadrics without splitting the facet of **L**. Such method is only possible when the given normals satisfy certain conditions. Under these conditions, this paper develops a local and quadratically precise method for the free-form surface problem using four quadric patches per facet. Quadratic precision is a measure of the accuracy of a method, meaning that if all the given data is taken from a quadric surface, the method produces the quadric surface; local means that each patch depends on the nearby given data only, so modifying a patch affects just the few neighbouring pathes. Along with the development of the method, the concept of smoothness is discussed. The usual notion of tangent plane continuity does not guarentee visual smoothness because the existance of cusps. We give a method for preventing cusps for implicit patches.

---

*This work is supported by DARPA under ONR contract N00014-86K-0591, NSF Grant DMC-86-17355, and ONR Grant N00014-89J- 1946.

1 The problem and its application

Free form surfaces are smooth surfaces which have to be approximated by piecewise algebraic surfaces. The approximations are done by interpolating a finite vertex set according to a given topology, which is described in terms of a piecewise linear interpolant **L** to the vertex set. Normal vectors, or normals as we shall call them, are often prescribed at the vertices to achieve control of the shape of the free-form surfaces. In this paper, **L** can be either open or closed, and the facets of **L** are triangular.

The free-form surface problem is a fundamental problem in geometric design and geometric modeling. Closed free-form surfaces are used for shape representation in geometric modeling; The automation of various industrial designs such as car bodies, airplanes, and ship hulls need appropriate methods for generating free-form surfaces. In addition, free-form surfaces are widely used to represent curved objects in computer graphics and robotics. [1].

2 Related work

Since most research on surfaces has focused on parametric form, a great deal has been done on the free-form surface problem in parametric form [11] [10]. Even though implicitly defined surfaces have been successfully used to solve many important problems [2] [3] [4] [5], its applications to the free-form surface problem are rarely seen.

The first attempt in using implicit surface in free form surface construction was made by Sederberg [6]. Taking up Sederberg's suggestion, Dahmen developed a method to solve the free-form surface problem [7], and his method is the most successful work so far on the free-form surface problem. Under the certain conditions on the given normals and most importantly, the condition that there exists a so called transversal system for **L**, Dahmen's method solves the free-form surface problem by putting 12 quadric patches on each facet of **L**. In comparison, our method solves the same problem with four quadric patches per facet. Moreover, the existance of a transversal system, which on one knows how to construct yet, is not assumed.

3 Rationale for using quadrics

In this paper. free form surface constructions are done with implicitly surfaces of the lowest possible degree. We choose implicit surfaces for the following reasons. First. the set of implicit surfaces properly contains all the parametric surfaces. so results on implicit surfaces are more applicable. Second. implicit surfaces have advantages over parametric surfaces in many applications: in geometric modeling. implicit surfaces has more compact storage and are closed under most operations; in computer graphics, many calculations such as ray tracing are easier when implicit surfaces are used. Third. so much work on the free-form surface problem is done parametrically. giving the impression that implicit surfaces are less suitable for the free-form surface problem. Our work shows the impression is not true.

From the statement of the free form surface problem. it is evident that the lowest degree possible to solve the problem is two. Increasing the degree does give more freedom in controling the shape of . However. the difficulties of dealing with surfaces increase rapidly with the the increase of the degree. Additionally. low degree implicit surfaces are easier to parameterize. So it is important to keep the degree low.

4 The comatibility problem

In this paper, we concentrate on developing free-form surface method that does not splitting the facets of **L**. An obvious reason for not allowing splitting is that splitting increases the number of patches needed for each facet. However. a more important reason for not allowing splitting is that to split a general piecewise linear surface requires the existance of the a transversal system [7]. which no one knows how to construct yet. So for each given piecewise linear surface, one has to manually creat the transversal system. This is against the general philosophy of computer applications.

So we set the goal of developing a method that solves the free-form surface problem using quadrics without splitting the facets. Unfortunately. such a method is impossible for arbitary given data: there are certain compatibility conditions have to be satisfied by the given normals. We shall derive these conditions and develop a method for the free-form surface problem under the conditions.

5 The use of geometric continuity

Our substantial improvement of the previous work is attributed to the application of geometric continuity. Geometric continuity has been successfully applied to the construction of parametric surfaces [8] and implicit blending surfaces [3]. However, little has been done to incorporate the idea of geometric continuity into the constructions of implicit free-form surfaces. All the existing work, such as [7] and [6], contour a C^1 function in three dimension. Such practices lead to the misleading impression that the implicit surface constructions are just contour of parametric forms of one dimesion higher so no separate studies are necessary. The surfaces produced by our method cannot be generated by contouring a C^1 function in $\mathbf{R}^3$. As a matter of fact, geometric continuity is explicitly used in the development of our method.

6 Real space vs complex space

Algebraic techniques play an important role in the research of implicit patches. Many algebraic theories are developed on a algebraic complete field, **C**, rather than **R**. As a result, most of the existing work using implicit patches has the following undesirable feature. The surfaces involved has complex coefficients, hence are not useful for representing physical objects. In this paper, the free-form surfaces are constructed exclusively with polynomials in $\mathbf{R}[x,y,z]$.

7 Cusps and a solution

In [8], DeRose pointed out that two surface patches can be tangent along a curve and still not be smoothly connected, i.e. the two surface patches form a cusp along their curve of intersection. The concept of oriented tangent plane continuity is used to avoid cusps for parametric surfaces. For algebraic surface patches, the cusps problem is more difficult. In fact, the problem that whether implicit surfaces are inferior when solving the cusps problem was brought up in [3]. We show that implicit surface is equally capable of dealing with cusping by giving a method for preventing cusps for implicit surfaces.

Acknowledgement: I am grateful to John Hopcroft for his comments and advices.

References

[1] J. Hopcroft, D. Krafft: *The Challenge of Robotics for Computer Science* . Advances in Robotics, Vol. 1: Algorithmic and Geometric Aspects of Robotics, edited by C. Yap and J. Schwartz, 1986.

[2] C. Hoffman, J. Hopcroft: *The Geometry of Projective Blending Surfaces*. Artificial Intelligence 37 (1988) 357-376.

[3] J. Warren: *On Algebraic Surfaces Meet With Geometric Continuity*. Ph.D. Thesis, Cornell, 1986.

[4] C. Bajaj: *Hermite Interpolation Using Real Algebraic Surfaces*, ACM Symposium on Computational Geometry, 1989.

[5] V. Pratt: *Direct Least Square fitting of Algebraic Surfaces*, SIGGRAPH '87.

[6] T. Sederberg: *Piecewise Algebraic Surface patches*, CAGD 2, 53-59.

[7] W. Dahmen: *Smooth Piecewise Quadric Surfaces*, Mathematical Methods in CAGD, edited by T. Lyche and L. Schumaker, pp. 181-193.

[8] T. Derose: *Geometric Continuity: A Parametrization Independent Measure of Continuity for CAD*, Ph.D. Thesis, Berkeley, 1985.

[9] P. Fjallstrom: *Smoothing of Polyhedral Models*, ACM Symposium on Computational Geometry, 1986.

[10] W. Boehm, G. Farin, and J. Kahmann: *A Survey of Curves and Surface Methods in CAGD* , CAGD 1, 1-60.

[11] J. Peters: *Local Smooth Surface Interpolation: A Classification*, University of Wisconsin, Preprint.