Planar Conflict Resolution for Air Traffic Control*

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1 Introduction and Problem Definition

Motion planning with moving objects is computationally harder than with stationary objects. Here such a problem arising in Air Traffic Control (ATC) is studied. The goal of ATC is to direct aircraft safely to their destinations while minimally interfering with their intended trajectories. This involves *maintaining* a minimum separation distance d (usually 5 nautical miles(n.m.)) between aircraft (considered as moving points), *detecting* conflicts (when the distance between aircraft is less than d), and *resolving* the conflicts according to certain maneuvering priorities and constraints.

Conflict Resolution being an instance of the *Asteroid Avoidance Problem* is an NP-hard problem. Resolution methods have been proposed which resolve the conflicts **subset by subset**, leading to backtracking and very high complexities. Such schemes can also lead to worse conflicts than the ones they resolved and pose some fundamental difficulties in program proving.

In this paper, we provide a new approach using a normalized relative coordinate model to both detect and resolve the conflicts. For each maneuvered aircraft, we represent it as three pseudo-aircraft indicating the status

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of that aircraft in different time periods (See Figure 1 and Table 1). For the multi-aircraft case, the resolutions obtained by the algorithm depend on the input sequence order. Here, heuristics are used to fix the order in the algorithm, whose time complexity is $O(N^2)$ without backtracking.

Even though the general problem is 3-dimensional in space, it is preferable, when possible, to resolve conflicts without altitude changes. For this reason the 2-dimensional case is studied first. The constraints for the evasive maneuvers are :

- maintain **constant speed** before, during and after the maneuver,
- execution of maneuver should be no earlier than some specified **lead time**,
- there is a maximum allowable **turn angle**,
- there is a maximum allowable deviation from the original track (**offset**), and
- upon completion, the aircraft returns to its original **heading** (see Figure 1).

The requirement for the *lead time* is due to the uncertainty in the radar tracking and also for avoiding unnecessary maneuvers in case the aircraft were going to turn anyway. Returning to the same heading facilitates maintaining course using directional radio navigation. It is difficult to control speed variations and for this reason constant speed maneuvers are preferred.

2 Method

2.1 Normalized Relative Coordinates

A moving circle with radius $d/2$ centered at each airplane is considered. Maintaining the minimum separation is equivalent to preventing these circles from intersecting (though they may touch). Starting with a pair of aircraft one circle is shrunk to a point (referred to as a moving point) and denoted by AC_i and the other, for aircraft AC_j, is doubled in radius without loss in generality.

The information is next transformed to a *normalized* relative coordinate system, in the sense that the information about all circles and their relative velocities with respect to a specific aircraft is presented on the **same diagram**, see Figure 2.

2.2 Constraints

To resolve the conflict, first, the *lead time* constraint is considered by translating the position of AC_j in the negative X direction. To avoid the conflict,

$\vec{V}_{ij}$ needs to be outside the conflict range $(\theta_{j1}, \theta_{j2})$ (see Figure 3). This may be accomplished by a rotation equivalent to changing the direction of AC$_i$ and preserving the magnitude $\left|\vec{V}_i\right|$, so as to satisfy the equal speed requirement.

When the maneuver starts, the course of AC$_i$ is updated to that of pseudo-aircraft AC$_{i'}$ (see Figure 1). Suppose that φ_i is chosen as the turn angle of AC$_i$ causing $\vec{V}_{ij}$ to rotate by some angle ψ_{ij}, then
$\varphi_i = \angle\vec{V}_{i'} - \angle\vec{V}_i \iff \psi_{ij} = \angle\vec{V}_{i'j} - \angle\vec{V}_{ij}$ ($\angle\vec{V}$ means "the angle of $\vec{V}$").

The strategy for satisfying the offset constraint is shown in Figure 4.

In effect, as shown in Table 1, AC$_i$ successively "spawns" 2 *pseudo-aircraft*, AC$_{i'}$ and AC$_{i''}$ (indicating the portion after the manuver), for specified time periods.

Table 1. The Information of Three Pseudo-Aircraft

Aircraft ID	Speed	Heading	Existing Time
AC$_i$	$\lvert\vec{V}_i\rvert$	$\angle\vec{V}_i$	$[\ 0, T_{i1}\ )$
AC$_{i'}$	$\lvert\vec{V}_i\rvert$	$\angle\vec{V}_i + \varphi_i$	$[\ T_{i1}, T_{i2}\ )$
AC$_{i''}$	$\lvert\vec{V}_i\rvert$	$\angle\vec{V}_i$	$[\ T_{i2}, \infty\ )$

T_{i1} : the turn time of AC$_i$; lead time before the first conflict.
T_{i2} : the earliest turn-back time satisfying all of the constraints.

2.3 Multi-Aircraft Resolution

For the multiple aircraft situations, the two-aircraft resolution algorithm is extended, so that the moving point avoids all other circles (including the ones due to the pseudo-aircraft), by using the normalized relative coordinate system. And, then, all of the aircraft are resolved one by one, using this extended model.

An example on realistic data provided by the FAA is shown in Figure 5. The ordering of the input sequence is pre-decided by a heuristic resulting in an $O(N^2)$ algorithm. Hence the time complexity of the whole algorithm is bounded by $O(N^2)$ and the space complexity is $O(N)$.

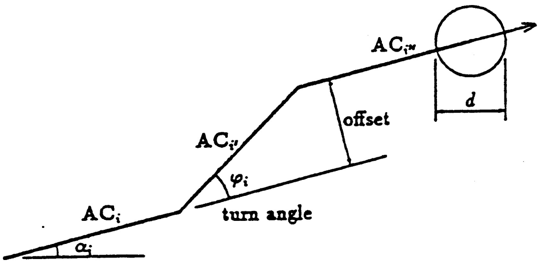

Figure 1. Notation for Aircraft i with Maneuver

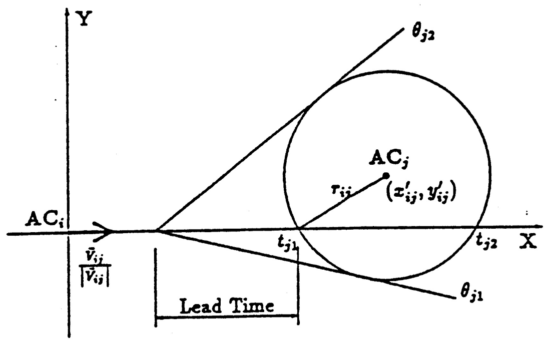

Figure 2. Conflict Angle Range of $\bar{V}_{ij}$

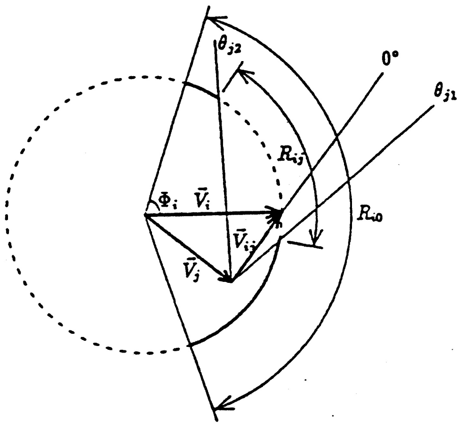

Figure 3. Conflict-free Turning Angle Range

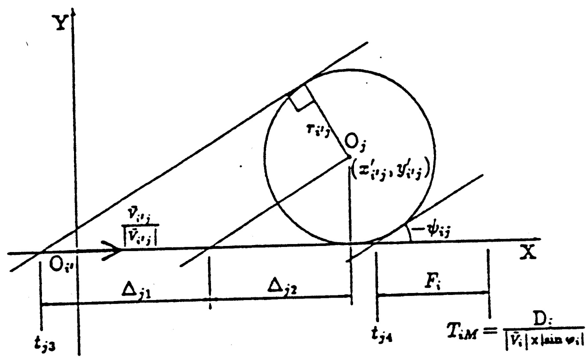

Figure 4. Conflict-free Turn-back Period

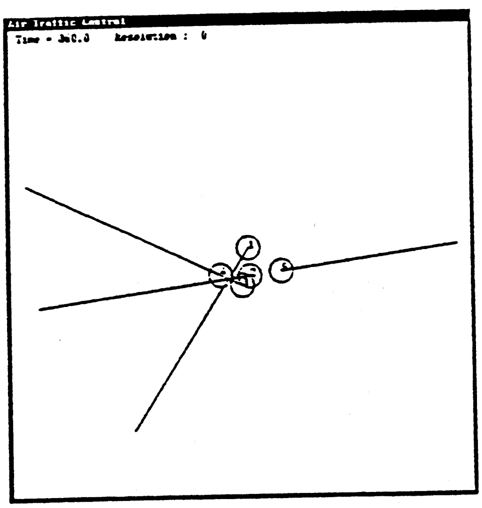

Figure 5(a). 5 Aircraft Unresolved

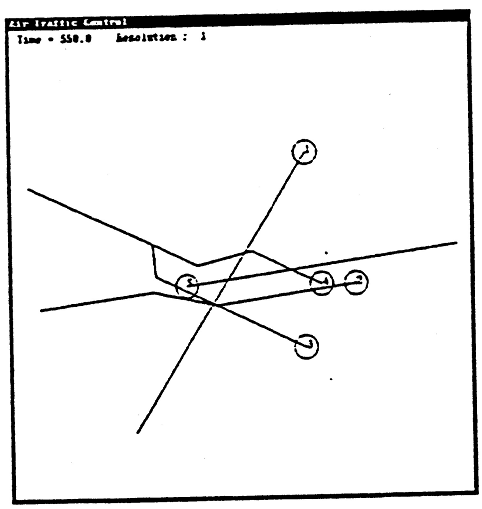

Figure 5(b). Resolution of 5 Aircraft