Dexterous Rotations of Polygons

(extended abstract)

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1 Background

Multifingered articulated hands have been proposed and studied by many members of the robotics community. By introducing dexterous hands as replacements for two stick end effectors, we seek to augment the manipulative capacity of robots with fine position and force control, thus increasing the reliability and complexity of task performances.

The effort to understand dexterous hands has been mostly concentrated in the area of grasping [Ngu86,MSS87,Bas88]. Grasps have been analyzed with respect to the number of fingers involved, the type of contact between the fingers and the objects, and such properties as equilibrium, stability, force closure, *etc.* In [Ngu86], Nguyen proposes an algorithm to synthesize force closure, stable grasps. However, little has been done to study what a mechanical hand can do with a securely gripped object. In [Mas82], Mason looks at the manipulation of objects through pushing and in [Bro87], Brock provides a framework for studying object manipulation through controlled slip.

We are interested in developing algorithms for automated manipulation. We define the manipulation of an object by a mechanical hand to be the reorientation of the object by some degrees about some axis. In the process, the hand never lets go of the object. One way of doing such rotations is by using a hand with a revolving wrist. Although this is a very simple solution to the problem, it is far from being sufficient. The main reason is that a revolving wrist only gives us rotations about one (say, z) axis. Therefore, we will focus on solutions that exploit the multifinger structure of mechanical hands. The rotations will be accomplished by finger motions rather than by a wrist motion.

1.1 Assumptions

The results presented here are for polygonal objects, robot hands with three or four fingers and frictionless point contacts between the fingers and the object. We are currently working on extensions to polyhedra, in the presence of friction.

We would like to accomplish the rotation of our object by starting with a *force closure* grip of the object and then by generating new *force closure* grips. A grasp is called *force closure* if an arbitrary force can be exerted on the object through the set of contacts. This is equivalent to saying that the fingers can resist any force applied on the object.

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2 The rotation of a polygon

2.1 Preliminary lemmas

For now, we restrict ourselves to triangles. The generalization will be presented in section 2.3.

Notational conventions: Let $\triangle$ ABC be a triangle and f_1, f_2, f_3 a set of fingers contacting BC, AB, AC at M, P, Q, respectively. In addition, we ask that $f_1 \perp BC, f_2 \perp AB, f_3 \perp AC$ and that $\exists G = f_1 \cap f_2 \cap f_3$, the intersection of the finger force directions. These requirements allow us to apply our results to gripping with point contacts in a frictionless environment. We call G *the center of the grip.* Let P and Q be two points fixed in the plane such that $\triangle ABC$ is constrained to maintain continuous contact with P and Q. We associate the vectors p_1, p_2, p_3, q_1, q_2 to the points C, B, A, Q, P in R^2, in some coordinate system. See *figure 1.*

Lemma 1: $\triangle ABC$ moves such that the vertex A and the center of grip G rotate on the unique circle defined by A, P, G and Q. No other motion is possible for the triangle. The transformation which represents this motion is of the form

$$T = \begin{pmatrix} cos\theta & sin\theta & a \\ -sin\theta & cos\theta & b \\ 0 & 0 & 1 \end{pmatrix}$$

Here, θ is the rotation angle with rotation matrix R_θ and $u = (a, b)^T$ is the unique associated translation, given by:

$$u = -\frac{det(R_\theta q_2, q_2)}{det(q_2, q_1)} q_2 + \frac{det(R_\theta q_1, q_1)}{det(q_1, q_2)} q_1$$

Corollary 1.1: The instantaneous center of rotation is the center of the grip.

Lemma 2: Let h_A be the height of the triangle at A and let D be the intersection of the force direction f_1 with $circle(A, P, G, Q)$. M moves on a circle of center D and radius equal to the length of h_A.

Corollary 2.1: Given a triangle and two fixed points in space which slide on two of the triangle's edges and a third point fixed on the other edge, the motion of the triangle is unique, and it is equivalent to the composition of a fixed rotation and some fixed translation. If f_1, f_2, f_3 are the respective force directions at the points of contact, then the triangle is fixed if $\exists G = f_1 \cap f_2 \cap f_3$; the triangle moves clockwise if the contact f_3 lies to the left of the perpendicular drawn from the intersection point of f_1 and f_2 onto the respective edge, and moves counterclockwise otherwise.

Lemma 3: Let $\triangle ABC$ be a triangle as before, with $\hat{A}$ acute and let f_3 be the finger pushing on BC. If P and Q are such that $| PQ | < h_B$ and f_3 causes clockwise rotations, or if P and Q are such that $| PQ | < h_C$ and f_3 causes counterclockwise rotations, then the triangle reaches a configuration in which it is blocked, *i.e.* f_3 can no longer cause motion by pushing.

2.2 Algorithm for rotations of triangles

These geometric results suggest ideas for algorithms to control the motion of a triangle by controlling the tracking of a finger on an edge. The main idea is that by keeping two fingers fixed and by displacing the third, the three force directions no longer meet at a point and the grip is no longer force closure. This causes the triangle to move until all three force directions converge to the same intersection point. As the triangle rotates, all the contact points slide on their respective edges. The rotation can continue until one of the fingers is about to slide off its edge, or until the triangle can no longer move. At such a point, it is possible to continue the motion of the triangle by assigning one of the previously fixed fingers to push, and by making fixed the finger that pushed previously. Thus, the range of the rotation angle can be increased by changing the pair of fixed fingers. In addition, we want our algorithm to be robust. In our framework, robustness means the ability to handle the uncertainties of the real world, by not requiring *a priori* knowledge of the geometry of the object, or exact arithmetic to determine the positions of the fingers. One strategy that reduces these knowledge requirements is *sensing*. By adding sensing capabilities to the fingers, some of the calculations can be replaced by contact sensing. *Lemma 3* provides a condition for robustness. We notice that such an algorithm for rotations of angles $> 2\pi$ performed by a three finger hand requires complete revolutions of the fingers. To bound the finger motions, we use a four finger hand.

Algorithm: Given a triangle, four fingers, and a desired reorientation angle, a rotation can be performed as follows. See *figure 3*.

1. Each edge of the triangle is assigned to one of the four fingers. The other finger is *free*. The rotation angle is initialized to zero.

2. Two of the assigned fingers are selected to be *fixed* and the other one to *push*.

3. The pushing finger slides in the same direction as the desired motion, thus causing a rotation according to *lemma 1* and *corollary 2.1*. Update the total rotation angle.

4. Reassign the fingers. If in the i^{th} iteration we have that $f_{i mod 4}$ is free, $f_{(i+1) mod 4}$ and $f_{(i+2) mod 4}$ are fixed and $f_{(i+3) mod 4}$ pushes, then for the $(i+1)^{th}$ iteration $f_{i mod 4}$ pushes, $f_{(i+1) mod 4}$ is free and $f_{(i+2) mod 4}$ and $f_{(i+3) mod 4}$ are fixed. With these assignments, repeat *step 3*, until the amount of rotation matches the desired rotation amount.

The robust version of this algorithm differs only in *step 3*. If the conditions of *lemma 3* are met, instead of precalculating the amount of slip for the pushing finger, the finger simply pushes until it senses that the triangle no longer moves, *i.e.* until it senses that the triangle has reached a blocking configuration.

2.3 Rotations of polygons

The algorithm developed for a triangle generalizes to polygons. The main idea for the generalization is that for any convex polygon, except for rectangles, by extending three of the polygonal edges, we obtain a minimal triangle which contains the polygon. For any convex polygon, except for rectangles, there is at least one such triangle, and possibly more. The larger the number of edges of the polygon, the larger is the number of minimal containing triangles.

Let Π be a polygon with edges $e_1, e_2, ..., e_n$. The following definitions and lemmas are needed to extend the result from the case of acute triangles to a decision procedure for arbitrary convex polygons.

Definition 1: A set of *candidate blocking regions* is a set of three *maximal* edge regions $\{r_1 = (U_1, U_2), r_2 = (V_1, V_2), r_3 = (W_1, W_2)\}$ such that $\exists e_i, e_j, e_k$ distinct with $r_1 \subseteq e_i$, $r_2 \subseteq e_j$, $r_3 \subseteq e_k$ and $U_1V_1 \perp e_i$, $U_2V_2 \perp e_i$, $V_1W_1 \perp e_k$ and $V_2W_2 \perp e_k$. See *figure2*.

Definition 2: A *candidate blocking distance* is the length of any line parallel to U_1V_1 in the trapezoid $U_1U_2V_2V_1$. The range of candidate blocking distances is denoted by Δ_{r_1,r_2}.

The candidate blocking regions represent locations of the fingers for which the polygon could be in a blocking configuration. That is, if $f_1 \in r_1, f_2 \in r_2$ and $f_3 \in r_3$, and the contacts are at P, Q and R and $PQ \perp r_1$ and $QR \perp r_3$, then the polygon is in a blocking configuration.

Notice that the condition given in the definition captures the ideas of *lemma 3* for the case of a polygon. In particular, r_1, r_2 and r_3 exist if $\widehat{e_i, e_j}$ is acute. The candidate blocking distances represent the range of the distances between the fixed fingers that yield blocking configurations.

Lemma 4: Let Π be a polygon and $\{r_1, r_2, r_3\}$ a set of candidate blocking regions. If d is a candidate blocking distance, and the fixed fingers are on e_i and e_j at distance d apart and the third finger pushes on e_k, then Π reaches a blocking configuration.

Lemma 5: Let Π be a polygon gripped in a blocking configuration corresponding to the candidate blocking regions $\{r_1, r_2, r_3\}$ on edges e_i, e_j, e_k. If there is a candidate blocking configuration $\{r_2, r_3', r_4\}$ on edges e_j, e_k, e_l and $\Delta_{r_2,r_3} \cap \Delta_{r_2,r_3'} = \Delta_\cap \neq \emptyset$ and if the distance between the fingers on e_j and e_k is in the range of $\Delta_\cap$, then the polygon can be moved from the initial (blocked) gripped configuration to a new blocked configuration by keeping the fingers on e_j and e_k fixed and by pushing on e_l.

Theorem: A polygon can be rotated robustly, if it is possible to iterate the conditions in *lemma 5* until there is a repetition of the edges supporting candidate blocking regions. If the polygon has n edges, then a repetition occurs after at most n iterations.

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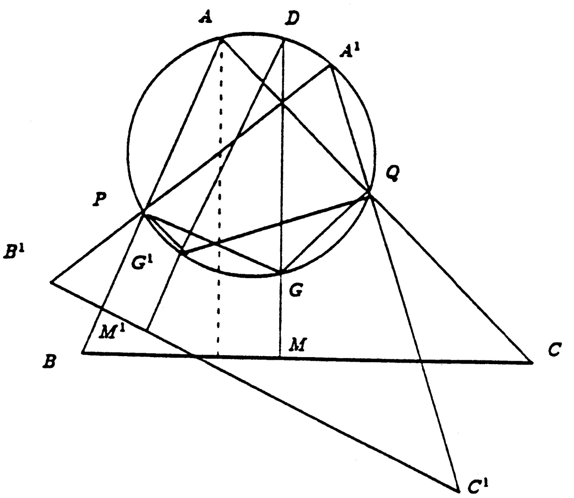


Figure 1: Construction illustrating the triangle lemmas

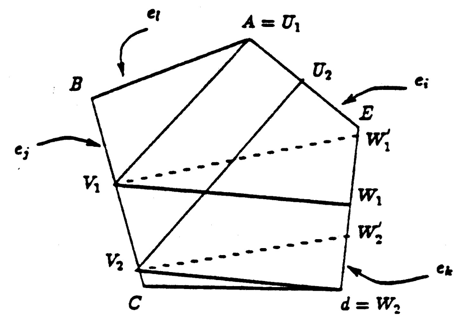
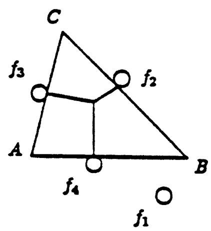
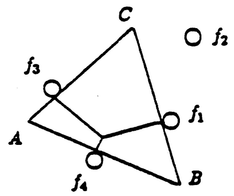


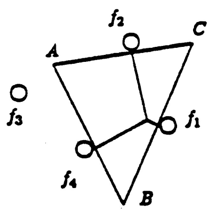
Figure 2: Construction of the candidate blocking regions



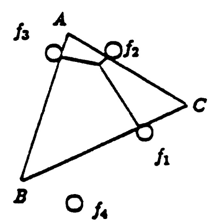
Initial configuration:
f_1 is free; f_4 pushes;



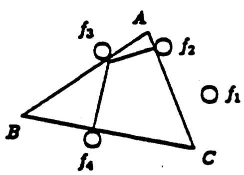
After the first step:
f_2 is free; f_1 pushes;



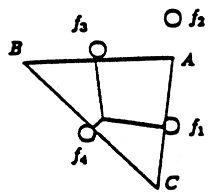
After the second step:
f_3 is free; f_2 pushes;



After the third step:
f_4 is free; f_3 pushes;



After the fourth step:
f_1 is free; f_4 pushes;



After the fifth step:
f_2 is free; f_1 pushes;

Figure 3: Five iterations of the algorithm