# A NOTE ON SEPARATION OF PLANE CONVEX SETS

( Extended Abstract )

by

Eduardo Rivera-Campo

## 1.- Introduction.

A line $l$ separates a set $A$ from a collection $S$ of plane sets if $A$ is contained in one of the closed halfplanes defined by $l$, while each set in $S$ is contained in the complementary closed halfplane.

H. Tverberg [1], proved that for every positive integer $k$, there is an integer $N(k)$ with the following property: If $F$ is a collection of $N(k)$ or more convex sets in the plane with pairwise disjoint relative interiors, then there is a line that separates a set in $F$ from a subcollection of $F$ with at least $k$ sets.

In this note, we show that in any family $F$ of $n \geq 2$ plane convex sets with pairwise disjoint relative interiors, there is a pair of sets A, B such that every line that separates A from B, separates either A or B from at least $\lceil n+58/60 \rceil$ sets in $F$.

## 2.- Main Results.

We start with a lemma which we state here without proof.

**Lemma 1.-** Let $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ be convex sets in the plane with pairwise disjoint relative interiors. If for each pair of integers i, j, with $1 \leq i < j \leq 5$, $l_{ij}$ is a line that separates $A_i$ from $A_j$, then for some pair s ,r, there is a set $A_t$ with $s \neq t \neq r$ such that $l_{sr}$ does not meet the interior of $A_t$.■

The main result in this note can now be presented.

**Theorem 2.-** Let $F = \{ A_1, A_2, \ldots, A_n \}$ be a collection of convex sets in the plane with pairwise disjoint relative interiors. For each pair i, j, with $1 \leq i < j \leq n$, let $l_{ij}$ be a line separating $A_i$ from $A_j$. There is a pair s, r, such that the line $l_{sr}$ separates either $A_s$ or $A_r$ from at least $\lceil n+58/60 \rceil$ sets in $F$.

**Proof.-** Define a bipartite graph $G$ as follows: There is a vertex $u_k$ in $G$ for each set $A_k$, and a vertex $v_{ij}$ for each line $l_{ij}$. A vertex $u_k$ is adjacent, in $G$, to a vertex $v_{ij}$ if and only if $l_{ij}$ does not meet the interior of $A_k$.

Using lemma 1 and the fact that for each pair i, j, with $1 \leq i < j \leq n$, the line $l_{ij}$ does not meet the interiors of $A_i$ and $A_j$, one can see that the number of edges in $G$ is at least $\binom{n}{5} / \binom{n-3}{2} + \binom{n}{2} = n(n-1)(n+58)/60$. This implies that the average degree of the vertices $v_{ij}$ is at least $\lceil n+58/30 \rceil$. Choose any vertex $v_{sk}$ in $G$ with degree at least $\lceil n+58/30 \rceil$; the corresponding line $l_{sk}$

does not meet the interior of at least $\lceil n+58/30 \rceil$ sets in $F$. It is clear that $l_{sk}$ separates either $A_s$ or $A_k$ from at least $\lceil n+58/30 \rceil/2 = \lceil n+58/60 \rceil$ sets in $F$. ■

As a corollary, we obtain the following result:

**Corollary 3.-** In any collection $F$ of $n$ plane convex sets with pairwise disjoint relative interiors, there is a pair of sets A, B, such that any line separating A from B, separates either A or B from at least $\lceil n+58/60 \rceil$ sets in $F$.■

## 3.- Acknowledgements.

I thank A. Bravo and J. Czyzowicz for the discussions we had on the proof of lemma 1. Corollary 3 is due to J. Czyzowicz.

## 4.- References.

[1] H. Tverberg, "A separation property of plane convex sets", Math. Scand. 45 (1979), 255-260.

Departamento de Matemáticas
Universidad Autónoma Metropolitana-Iztapalapa
Av. Michoacán y Purísima, Col. Vicentina
Apdo. Postal 55-534, C.P. 09340
México D.F., México