Computing Bushy and Thin Triangulations

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Abstract

Given a triangulation of a polygon P, let t_2 be the number of leaves in the dual tree of the triangulation. Also, define $t_{max}(P)$ and $t_{min}(P)$ as the maximum and minimum values of t_2 over all triangulations of P. We let $T(n)$ represent the time complexity of computing the triangulation of a polygon with n vertices.

We call a triangulation of P *bushy* if it has $t_2 = t_{max}(P)$, or *thin* if it has $t_2 = t_{min}(P)$.

By using an $O(T(n))$ method for identifying the *ears* (possible leaves in the dual tree) of a polygon, we arrive at an $O(T(n))$ time, $O(n)$ space algorithm for finding a bushy triangulation. We show that for monotone, star-shaped, or edge-visible polygons, we can find a bushy triangulation in $O(n)$ time. Furthermore, we show that one can also find a bushy triangulation in $O(n)$ time for palm polygons with a known palm point. Thus, the problem of computing a bushy triangulation seems to be of the same complexity as computing any triangulation.

We use dynamic programming to achieve an $O(n^3)$ time, $O(n^2)$ space algorithm for finding a thin triangulation.

Finally, we note that the problems of computing bushy and thin triangulations are optimization problems (over all triangulations of the given polygon) on some combinatorial measure of the triangulation. We present several other related problems of this sort, in the form of other interesting measures of polygon triangulations.