On Vertical Visibility in Arrangements of Segments and the Queue Size in the Bentley-Ottman Line Sweeping Algorithm

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Abstract

Let $S = \{e_1, e_2, ..., e_n\}$ be a collection of n (intersecting) line segments in the plane. We will show that the number $\mu(S)$ of non-intersecting pairs (e_i, e_j) that can see one another vertically is $O(n^{4/3}(\log n)^{2/3})$, and can be $\Omega(n^{4/3})$ in the worst case. However, if we assume that all segments have their right end points lying on the same vertical line and count only pairs of non-intersecting vertically visible segments that will intersect when extended to the right, then there are at most $O(n\log^2 n)$ such pairs, and only $O(n\log n)$ in the case of full rays, where the latter bound can be obtained the worst case. We apply these results to obtain similar upper and lower bounds on the maximum size of the queue in the original implementation of the Bentley-Ottman algorithm for reporting all intersections between the segments in S, i.e. when future events are not deleted from the queue.