Algorithms for Computing the Center of Area of a Convex Polygon

Matthew Díaz[†] & *Joseph O'Rourke*[*]

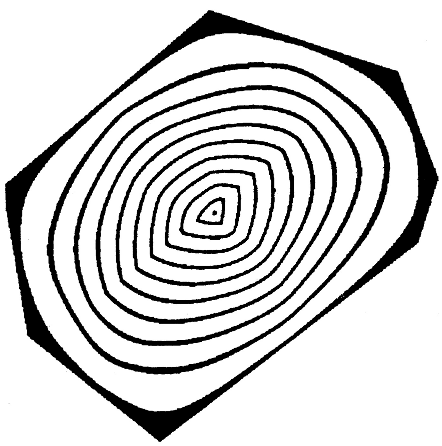
†Department of Computer Science, The Johns Hopkins University, Baltimore, MD, USA, 21218

*Department of Computer Science, Smith College, Northampton, MA, USA, 01063

Abstract

The center of area of a convex polygon P is the unique point p^* that maximizes the minimum area overlap between P and any half-plane that includes p^*. More formally, let $\alpha(p)$ be the minimum area intersection between P and any half-plane that includes p. Then the center of area of a polygon is the set of points that maximize $\alpha(p)$ over all points p in the plane. We show that the center of area is a unique point p^* and derive related geometric properties.

We present both a combinatorial and a "numerical" algorithm for computing the coordinates of p^* for a polygon with n vertices. The combinatorial algorithm, which runs in $O(n^6 \log^2 n)$, assuming exact real computations, is intended solely to demonstrate that the solution can be found in polynomial time. The second algorithm is numerical in the sense that we have been careful to express the complexities in terms of G, the number of bits used to represent the coordinates of the polygon vertices (the size of a "grid" on which the polygon can be drawn) and K, the number of desired bits of precision in the output. This algorithm runs in $O(nGK)$ and permits a straightforward implementation. Results for non-convex and orthogonal polygons have also been derived.



The center of area and level surfaces (contours) of $\alpha(p)$ are shown for the polygon P. (The contour for a value δ is defined as $\{p : \alpha(p) = \delta\}$.) The contours shown are taken at five percent intervals with each contour extending over the range $[\delta, \delta+1)$ percent. Thus the outermost contour is for the range $[0, 1)$ percent. The value at the center of area, $\alpha(p^*)$, is 48.92% of the total area.