An Incremental Algorithm for the Farthest Voronoi Diagram

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Abstract

The farthest Voronoi diagram is a variation of the standard Voronoi diagram. For a given set of points $P_i (i = 1, ..., n)$, we call them generaters, the farthest Voronoi region of P_i is the set of points whose 'farthest' generater is P_i, while the standard Voronoi region of P_i is the set of points whose 'nearest' generater is P_i.

Here we propose an incremental algorithm which constructs the farthest Voronoi diagram in $O(n_c)$ (n_c : number of points on the convex hull of the given points) on the average if the convex hull of the given points are obtained. The average convexity of the Bentley-Shamos algorithm for the convex hull is $O(n)$, we can construct the farthest Voronoi diagram in $O(n)$ on the average where n is the number of given points.

The properties of the farthest Voronoi diagram which we use for our algorithm are as follows:
(1) Only the points on the convex hull have their farthest Voronoi regions.
(2) All the farthest Voronoi regions are infinite regions.
(3) The farthest Voronoi regions of the two neighboring points on the convex hull have a common Voronoi edge, and the edge is an infinite edge.
(4) The number of Voronoi edges is $2n_c - 3$, and the number of Voronoi points is $n_c - 2$.

We devise a simple ordering procedure for the incremental points to attain the average linear order of the algorithm. Starting with a three points farthest Voronoi diagram, we increment each point uniformly in our algorithm. We number the points on the convex hull on the counterclockwise order. First we make the initail Voronoi diagram for the 1st, $(\lfloor n_c/3 \rfloor)$th, and $(2\lfloor n_c/3 \rfloor)$th points on the convex hull. The points on the convex hull are divided by three parts, i.e. from the 2nd points to the $(\lfloor n_c/3 \rfloor - 1)$th, from the $(\lfloor n_c/3 \rfloor + 1)$th to the $(2\lfloor n_c/3 \rfloor - 1)$th, and from the $(2\lfloor n_c/3 \rfloor + 1)$th to the n_cth. Then, we select the incremental point from the three parts equally until all the points on the convex hull are incremented.

Our computational experiment shows that our algorithm constructs the farthest Voronoi diagram in $O(n_c)$ on the average.