

Two Algorithms for Radial Ordering of Delaunay Triangles

Leila De Floriani

Istituto per la Matematica Applicata del C.N.R., via L.B. Alberti, 4, 16132 Genova, Italy

George Nagy

Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, New York, 12180-3590, USA

Abstract

In a two-dimensional Delaunay triangulation T , there exists a partial order of the triangles with respect to a vertex v (called the *viewpoint*) that is consistent with the two-dimensional visibility of the triangles from that vertex (*star-shape property* of Delaunay triangulation). In such a partial order a triangle t' is considered to be *predecessor* of another triangle t'' of T when every subset D of triangles of T which defines a polygon R star-shaped with respect to v and contains t'' , contains t' as well. We show that a linear order L of the triangles of T consistent with such a partial order can be constructed from T in linear time, and we propose two different algorithms for computing such an order.

The first algorithm computes a linear order L from T and v starting with the triangles of T incident on v , and incrementally adds one triangle in such a way that the resulting polygon R defined by the triangles considered so far is star-shaped. At the beginning the triangles of T incident on v are inserted into L in any order and the current polygon R is initialized with the edges which belong to these triangles and are not incident on v . Then, for each edge e of R , the triangle t of T adjacent to R along e and exterior to R is considered: t can be added to R when either it has two edges on R or the vertex of t exterior to R lies in the sector defined by the two rays emanating from the viewpoint through the end points of e . If t does not satisfy either condition, then edge e is eliminated from further consideration. Triangle t will be added to L only after the insertion of one of those two of its three adjacent triangles that do not share edge e with t .

The second algorithm is based on a radial ordering of the edges of T derived from an equivalent formulation of the concept of predecessor of a triangle. A triangle t' of T is a predecessor of another triangle t'' of T when a ray from v that intersects the edges shared by t' and t'' enters t' before t'' . This defines a linear number of relations over the triangles of T , where each relation corresponds to an edge of T . A linear order of the triangles of T can thus be obtained by applying a topological sorting algorithm to the triangles of T with the relation defined above.