The Superman Problem

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Abstract

Given two simple polygons K and P with K in the interior of P and a point s outside P, a minimal obscuring set for K, P and s is the smallest subset of edges of P that hides K from s. We present a $O(T(n))$ algorithm that finds a minimal obscuring set for K, P and s, where $T(n)$ is the time required to triangulate a simple polygon of n sides. We refer to this problem as "the Superman Problem" (for reasons given in the presentation).

We perform a transformation from the Superman Problem to the Circle-Cover Minimization Problem: given a set of arcs on a circle, find the minimum number of arcs whose union covers the circle. Lee and Lee have given an algorithm that solves this problem in $O(n \log n)$ time, or $O(n)$ time if a crucial function (the "successor" function) is implemented in linear time. We show that the structure of the Superman Problem permits us to implement this "successor" function for the corresponding circle cover minimization problem in linear (amortized) time.

Our algorithm for the Superman Problem starts by removing those segments of P which could not possibly contribute to a minimal obscuring set: these are the segments that do not belong to the visibility polygon of s given K ($VP(s|K)$), or that do not hide any part of K from s. This is accomplished by traversing the subdivision formed by merging $VP(s|K)$ with an exterior triangulation of P. We then produce a *radial trapezoidization* of the remaining edges of P with respect to s. A *radial trapezoid with respect to point s* is a quadrilateral with two nonadjacent edges contained in rays with vertex s. A radial trapezoidization of a subdivision with respect to s is a further subdivision into radial trapezoids with respect to s. Finally, we apply Lee and Lee's circle-cover algorithm, using a *radial sweep line* in our radial trapezoidization to quickly implement the necessary "successor" function. The minimal obscuring set that we seek is shown to be the edges that correspond to the arcs in the minimal cover of the circle.