Rectilinear Hull - Efficient Sets - Convex Hull : Relationship and Algorithms

Dilip Chhajed and Vijay Chandru#*

*Krannert Graduate School of Management, Purdue University, West Lafayette, IN 47907.
#School of Industrial Engineering, Purdue University, West Lafayette, IN 47907.

Abstract

Consider the following seemingly unrelated problems:
Problem 1: Given a set S of n points, on the plane, find a smallest isothetic polygon containing S such that each pair of points in the polygon has a shortest rectilinear path contained in the polygon. We call this polygon, the Rectilinear Hull of S.
Problem 2: Given a set S of n points, on the plane, find the set of efficient, strictly efficient and alternately efficient points. A point x° is efficient if there exists no x which is as close to each $s_i \in S$ as x° and strictly closer to at least one s_i. An efficient point x° is said to be alternately efficient if there exists another point $x^\bullet$ which is at same distance from s_i (for each $s_i \in S$) as x°. Efficient points which are not alternately efficient are strictly efficient.

We show that Problems 1 and 2 are equivalent. In this paper we present unified algorithms for both problems. Our motivation for this work arises from the following related problem which has application in facilities layout:

Problem 3: Given a set S of n points, on the plane, and a set F of pairs of points, construct a network of minimal length, in which each pair of points in F is connected by a shortest rectilinear path.

In this paper we define rectilinear hull of a set of points S and give a combinatorial characterization which enables us to find rectilinear hull in polynomial time. We prove the equivalence between rectilinear hull and efficient set and give two algorithms for finding the rectilinear hull. The algorithms are based on the plane sweep and divide and conquer paradigms. Our way of representation allows an easy identification of the alternate and the strictly efficient sets. We also show that these algorithms are of O(nlogn) time complexity where n is the number of points in S. Finally, we show how to go from the rectilinear hull to the convex hull of a point set. We use this to show that the algorithms are optimal by showing that the time complexity of the rectilinear hull problem is also Ω(nlogn).