

TRANSLATION SEPARABILITY OF POLYHEDRA

DORON NUSSBAUM AND JÖRG-RÜDIGER SACK

SCHOOL OF COMPUTER SCIENCE, CARLETON UNIVERSITY, OTTAWA, CANADA K1S 5B6

ABSTRACT

The rapid development in the fields of computer graphics, CAD-CAM systems, and robotics has attracted attention to problems of moving objects, e.g. line segments or polygons, in two and three dimensional space. Here we are interested in a motion planning problem which appears in the context of separability motions, where one wishes to separate each object from a collection of objects via collision-free motions. In its most general form this problem has been shown to be P-space hard. One computationally feasible variant of this problem has received considerable attention; it assumes that all separability motions are translations which are performed one translation motion per object.

Different aspects of this problem class have been studied such as to compute sequences in which to perform the separation motions as well as to determine whether a given collection of objects can be separated via such motions. In this context, the authors have recently shown that detecting the existence of a separation sequence for a set of simple polygons in a specified direction is as hard as computing one and both have optimal solutions.

In this paper, we study the problem of determining directions of separability (via translation). In two dimensional space, this problem has been well studied. In three dimensional space the problem is more complex. Toussaint has given an $O(n^2 \log n)$ algorithm for the problem of computing all directions of separability of two convex polyhedra with n vertices each. Nurmi and al. have given an optimal, linear-time solution for this problem. Recently, also Natarajan has studied related problems.

Here, we present an $O(m^2 n^2 \log mn)$ algorithm for determining all directions in which two simple polyhedra with n and m vertices can be collision-free translated. By giving a construction (also discovered by Pollack et al.) one can see that the algorithm is nearly optimal. This $\Omega(m^2 n^2)$ lower bound still holds when two simple polygonal faces are separated in 3-space, we also give an $O(m^2 n^2 \log mn)$ upper bound for this case. The algorithms presented have some interesting applications.