ORDER MODELS FOR MOTION IN THREE-SPACE

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Abstract

To each member C_i of a family $C_1, C_2, \ldots$ of sets in R^n assign a direction d_i of motion. Each of these sets may represent a robot supplied with a direction of motion d_i along which it may be moved with some velocity to separate it from the others without collision. Say that C_j *obstructs* C_i if there is a point p_i in C_i and a line from p_i parallel to d_i which intersects C_j. More generally, C_j *blocks* C_i, write $C_i < C_j$, if there is a sequence $C_i = C_{i_1}, C_{i_2}, \ldots, C_{i_m} = C_j$ such that $C_{i_{k+1}}$ obstructs C_{i_k}, for each $k = 2, 3, \ldots, m$. This relation <, called a *blocking* relation, induces a(strict) order on the family of sets in R^n, as long as < contains no directed cycles (that is, < is antisymmetric). In contrast to its two-dimensional analoque [Guibas and Yao (1980), cf. Rival and Urrutia (1988)], there are families of closed convex sets in R^3 for which any assignment of directions, one to each set, induces a directed cycle, whence cannot be ordered at all [cf. Dawson (1984)].

On the other hand, an ordered set P is *representable* in R^n if there is such a collection of sets in R^n whose blocking relation is the same as P. We have shown elsewhere that there are ordered sets that are not representable in R^2 using only closed convex sets [Rival and Urrutia (1988)].

THEOREM 1. *Every ordered set is representable in* R^3.

An ordered set P has a *d-directional representation* in R^n if it is representable in R^3 using at most d directions. We have shown that there is a correspondence between one-directional representations in R^2 and planar embeddings of planar lattices [Rival and Urrutia (1988)].

THEOREM 2. *Not every ordered set has a one-directional representation in* R^3 *using closed, convex sets.*

In R^2, every one-directional representation (equivalently, any planar lattice) can be modelled using just line segments as the closed convex sets and, moreover, all may be taken parallel.

THEOREM 3. *Any one-directional representation in* R^3 *of a family of subtrees of a tree in is a dismantlable lattices. On the other hand, there are dismantlable lattices with no one-directional representation at all in* R^3 *using a family of subtrees of a tree.*

What is apparently too constrained in three-space becomes comfortably manageable in R^4.

THEOREM 4. *Every ordered set has a one-directional representation in* R^4 *using closed convex subsets.*

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