L^1 Embeddability, Complexity and Multicommodity Flows

David Avis

School of Computer Science, McGill University, 3480 University Street, Montreal, Quebec, Canada H3A 2A7

Michel Deza

CNRS, Université Paris VII, UA 212, 2, Place Jussieu, 75251 Paris, cedex 16, France

Abstract

A finite metric d (or more properly semimetric) is L^1–embeddable if it can be expressed as a non-negative combination of cut metrics. When d is rational this is equivalent to a multiple of d being isometrically embeddable in a hypercube of suitably high dimension. The cone of L^1–embeddable metrics is called the Hamming cone. In this talk we provide a unified setting for describing a number of results related to L^1–embeddability. We collect and describe results on the facial structure of the Hamming cone and the complexity of testing L^1–embeddability of a metric. We describe the role of such metrics in a number of areas including multicommodity flows, combinatorial optimization and measure theory. In particular we show how the specializations of the so-called Japanese Theorem, for the feasibility of multicommodity flows, to a Ford-Fulkerson type theorem depend critcally on the L^1–embeddability of certain small metrics. Finally we give some results on the facial structure of cones generated by subsets of cut metrics.