# Distance graphs in Euclidean space

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## Abstract

Let D be a non-empty set of real numbers. For a point set X in Euclidean n-space, the <u>D-distance graph</u> on X (denoted by X(D)) is the simple graph with vertex set X and edge set $\{xy:d(x,y) \in D,\ x,y \in X\}$. Specifying D in various ways, there arise many interesting graphs as X(D). For example, letting D = {1}, [0,1], N, A, S, we get the unit distance graph, the unit neighborhood graph, the integral distance graph, the algebraic distance graph, and the surd distance graph, on X, respectively. (A number is called a <u>surd</u> if it is obtained from 0 and 1 by applying finite times of arithmetical operations and extractions of square root.) Related to these graphs, many interesting results are obtained. Let us cite just one: For a finite point set X in n-space, the algebraic distance graph X(A) is "rigid" in n-space if and only if X(A) is a complete graph (Homma-Maehara).

After a brief survey on distance graphs, we construct a rigid, surd distance graph X(S) in the plane which is not complete. In other words, we present a rigid graph in the plane (with edges all line segments) which cannot be constructed by ruler and compass from the data of incidence relation and edge-lengths. Further, under certain conditions, we prove that if a distance set D has the property that for any finite set X in the plane, the rigidity of X(D) implies the completeness of the graph X(D), then D is a field containing the surd field.