Algorithmic and Combinatorial Techniques for Graphs in $\mathbb{R}^d$

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Abstract

One of the important concepts in combinatorial and computational geometry is that of spatial graphs, i.e. graphs defined for finite point sets in $\mathbb{R}^d$.

More specifically, let ρ be a relation from S to S where $S = \{p_1, \cdots, p_N\}$ is a point set in $\mathbb{R}^d$. By a spatial graph induced by a relation ρ we mean a graph $G(S, E)$ such that $\overline{p_i p_j} \in E$ if and only if $p_i \ \rho \ p_j$. Usually the relation ρ is defined using concepts such as distance or angle measure. Well-known examples include the nearest neighbor graphs, the relative neighborhood graphs, spanning trees, and repeated distance graphs.

In this paper we shall survey some of the highlights of the algorithmic and combinatorial techniques used to construct and analyze these types of graphs. We shall also discuss some applications of spatial graphs to other domains.

We can give only a selection of problems in this domain; specifically in this paper we will give a progress report regarding relative neighborhood graphs (*RNG*). Several examples and results will be given -- some classical and well-known, some less-known, and a few new.

Our objective is twofold;

- to present some of the flavor of the recent work with *RNG* graphs in $\mathbb{R}^d$
- to present (possibly) uniform techniques to construct *RNG* graphs efficiently

In particular we are planning to discuss relatively new algorithms which construct neighborhood graphs in $\mathbb{R}^d$ space in $o(N^3)$ time. Also, some findings regarding the size of relative neighborhood graphs in $\mathbb{R}^d$ will be given.