CONLFLICT RESOLUTION, ONE-SHOT PROBLEM & AIR TRAFFIC CONTROL

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The goal of Air Traffic Control is to direct aircraft safely to their destination while minimally interfering with their intended trajectories. This involves maintaining a minimum separation distance d between aircraft (considered as points), *detecting* conflicts (when the distance between aircraft is less than d) and *resolving* the conflicts according to certain maneuvering priorities. **Conflict Resolution** being an instance of the *Asteroid Problem* is an NP-Hard Problem. For two aircraft AC_k and AC_i the *front and back limiting trajectories* F_{ik} , B_{ik} are found. They are the trajectories of two points, moving at the same velocity V_k and the same initial horizontal coordinate x_1 as AC_k, which are tangent to a circle centered at AC_i with diameter $2d$, passing the circle in the front and back respectively. A parallelogram enclosing the circle AC_i is found. The representation of lines in R^N (here N=4) in parallel coordinates in terms of $N-1$ indexed planar points is used. Relative to AC_i for each other aircraft AC_k there is a vertical interval I_{ik} consisting of points representing *all* the paths between the front and back limiting trajectories. The interval T_{ik} consists of the times between when the two limiting trajectories enter the parallelogram of AC_i. Aircraft AC_i and AC_k are in conflict $\Leftrightarrow$ the point $[1:2]_k$ representing the path of AC_k (in the x_1x_2-plane) $\in I_{ik}$. and entering the parallelogram of AC_i at a time $t_k \in T_{ik}$. In that case the closest trajectory, with the same velocity, for AC_i is represented by the closest endpoint of the **union** of all intervals I_{ij} with the point, representing the path of AC_i, $[1:2]_i \in I_{ij}$ and $t_j \in T_{ij}$. This trajectory is free of conflicts with **all** other aircraft. The time available needs to be matched with the appropriate maneuvers for feasibility. The algorithm is constrained to place the aircraft on a new conflict-free trajectory only if it is within d distance from the original. Any *convex* shape for the protected airspace can serve just as well. To handle the problem of high complexity the algorithm operates with rules in 3 levels. The first two levels involve *one initial maneuver* (the first level without and the second level with some form of recursion) per aircraft while the *advanced rules* permit an arbitrary number of maneuvers. At present the complexity for level 1 rules is $O(N^2 \log N)$, level 2 $O(N^4 \log N)$ and 3 $O(qN^4 \log N)$ here N is the total number of aircraft and q is the maximum allowable number of maneuvers. The algorithm was checked on some FAA supplied complex scenarios. The algorithm is *parallelizable* and certain other extensions (e.g. velocity changes) look promising.

For a set of moving spheres in a space of arbitrary dimensionality, the search for a trajectory of a point moving with constant velocity which will hit **all** the spheres was recently shown to be in an *NP-complete* problem (One-Shot Problem). In our case this occurs when the intersection of all the vertical intervals i.e. $\bigcap_{i \neq k} I_{ik} \neq \emptyset$, any point in this intersection representing the trajectory of a point travelling with the same velocity as AC_i and which is also time compatible. Resolution methods have been proposed which resolve the conflicts **subset by subset**, leading to backtracking and very high complexity. Using solutions for the one-shot problem found in this way, it was shown that such **partial resolution** schemes can lead to worse conflicts than the ones they resolved in addition to posing some fundamental difficulties in program proving; hence the preference for the *global* resolution method proposed here. "One-shot patterns" also look very useful in understanding and classifying inherently dangerous configurations.

1