Computing Minimal Spanning Covers of Sets

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Abstract

Given a family of n sets $\boldsymbol{F} = \{F_1, F_2, \ldots, F_n\}$ and a covering object C, we say that C is a *spanning cover* of $\boldsymbol{F}$ if there exists a placement of C such that for all i=1,2,...,n we have $F_i \cap C \neq \emptyset$. The smallest covering object that is a spanning cover of $\boldsymbol{F}$ is called the *minimal spanning cover* of $\boldsymbol{F}$. The *spanning cover* problem generalizes and unifies several well known classes of problems previously considered in computational geometry such as determining *common transversals* and computing *reachability* regions in robotics. The *minimal spanning cover* problem on the other hand has been previously unexplored in almost all settings as for example in finding shortest transversal segments. In this paper we provide efficient solutions to several instances of this problem.

Given a simple polygon P consisting of n sides and a line L not intersecting P, we say that P is *weakly externally visible* from L if for every point x on the boundary of P there exists a point y in L such that the interior of the line segment [x,y] does not intersect the interior of P. Clearly a convex polygon is *weakly externally visible* from every such line L. However, it is not necessarily so visible from a given line segment. It is shown that, given a *convex* polygon P, the *minimal length* line segment from which P is *weakly externally visible* can be found in O(n) time. The algorithm is based on the solution to a geometric minimization problem that is of independent interest and should find application in several different contexts, i.e., finding the shortest transversal segment that intersects the two rays of a cone while remaining tangent to a polygon P contained in the cone. As one application of this problem we consider finding the shortest line segment that intersects a given set of n line segments and present an $O(n \log^2 n)$ time algorithm for obtaining a solution.