Ordered Stabbing of Pairwise Disjoint Convex Sets in Linear Time

Peter Egyed[*] and Rephael Wenger[†]

[*]School of Computer Science, McGill University, 3480 University Street, Montreal, Quebec, Canada H3A 2A7
[†]Département d'Informatique, Université de Montréal, C.P. 6128, succursale A, Montréal, Quebec, Canada H3C 3J7

Abstract

A *line stabber* or *line transversal* is a line which intersects every member of a family of objects. Let A be an ordered family of n pairwise disjoint convex simple objects in the plane. Objects are simple if their common tangents can be found in constant time. We show how to find a line stabber for A which is consistent with the ordering of A in $O(n)$ time. This linear algorithm contrasts with the $\Omega(n\log n)$ lower bounds for finding a line stabber for A which is not necessarily consistent with the ordering of A.

Our algorithm first finds a range of slopes such that a line stabber of A has slope within the range if and only if the line stabber intersects A consistent with the given ordering. The algorithm then attempts to find a line stabber for A within the range.

For each slope there exists a unique '*lowest*' line with the given slope such that the closed half-plane below the line contains one object in A and every object in A intersects the closed half-plane above the line. Similarly for each slope there exists a unique '*highest*' line with the given slope such that the closed half-plane above the line contains one object in A and every object in A intersects the closed half-plane below the line. By comparing these two lines, one can tell whether there exists a stabber for A with the given slope.

The algorithm constructs a representation of the 'lowest' line for each slope, by processing each element of A in order. A simple backtracking procedure corrects the representation of the 'lowest' lines as each element is processed. By using the ordered structure of the objects and the pairwise disjointness condition, we prove that the backtracking procedure correctly builds the representation in linear time. Similar steps are taken to construct a representation of the 'highest' lines. The two representations can be merged and compared in linear time.

This linear algorithm corresponds to a Helly-type theorem for line stabbing pairwise disjoint compact convex sets in order. An ordered family of pairwise disjoint compact convex sets has a line stabber consistent with the ordering if and only if every four convex sets have a line transversal consistent with the ordering.