An Optimal Algorithm for Shortest Rectilinear Paths Among Obstacles in the Plane

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Abstract

We present an optimal $\Theta(n \log n)$ algorithm for determining a shortest path according to the L_1 (L_∞) metric between a source and a destination in the presence of disjoint polygonal obstacles in the plane. Our algorithm requires only linear space to build a planar subdivision (a Shortest Path Map) such that the length of a shortest path from the source to any query point can be reported in time $O(\log n)$ by locating the query point in the subdivision. An actual shortest path from the source to any query point can be reported in time $O(k + \log n)$, where k is the number of "turns" in the path.

The algorithm uses a technique which runs much in the spirit of Dijkstra's algorithm. We propagate a "wavefront" out from the source, keeping track of "events" that occur when the wavefront makes critical changes. A crucial property that we are employing is the fact that the wavefront will be piecewise-linear, with segments of fixed orientations. This allows us to compute events by performing "segment dragging queries", which have been solved in optimal ($O(\log n)$) time and linear space by Chazelle. We do not keep track of the exact structure of the wavefront at any instant (since we do not know how to solve for the next event in such a case), but rather we allow the wavefront to "run over" itself in a controlled way. (Basically, we acknowledge only those events that occur when a segment of the wavefront collides with an obstacle, rather than checking for all collisions between one wavefront segment and another.)

The algorithm can be generalized to find shortest paths according to any "fixed orientation" metric, which yields an $O(\frac{n \log n}{\sqrt{\epsilon}})$ approximation algorithm for finding Euclidean shortest paths among obstacles. The algorithm can further be generalized to the case of multiple sources to build a Voronoi diagram for multiple source points which lie among a collection of obstacles in time $\theta(N \log N)$, where N is the maximum of the number of sources and the number of obstacle vertices.