High-Clearance Motion Planning for a Convex Polygon among Polygonal Obstacles

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Abstract

We develop a technique for high-clearance motion planning for a convex polygonal object moving among polygonal obstacles in the plane. We allow both rotation and translation (*general motion*). For a k-sided convex object and obstacles with n corners and edges, the time needed to determine a path is $O(k^4 n\ \lambda_4(kn)\ \log n)$, where λ_4 is one of the almost-linear functions related to Davenport-Schinzel sequences.

Some algorithms for the motion of a convex object have required that the object remain in contact with an obstacle as it moves. In practice, this type of motion is undesirable: small errors in the control of the motion can cause the object to crash into the obstacle or friction can cause the object to bind. The time bound for our technique (which produces high-clearance paths) is about the same as the best time-bound known for planning the motion of a convex polygon without high-clearance.

Our technique uses the intuitively appealing idea that motion along Voronoi boundaries provides maximum clearance. The difficulty is that the structure of the appropriate Voronoi diagram changes as the movable object rotates. If we plot the changing Voronoi diagram in (x,y,θ) space then we get a collection of cells in 3-space. We show that the Voronoi cell-complex is not large by using an analysis technique developed by the authors to bound the number of changes that can take place in a Voronoi diagram as the distance-defining convex shape is rotated.

We construct a graph that contains all the information necessary to do high-clearance motion planning. The graph is a *skeleton* of the Voronoi cell-complex described above; it includes all the edges of the cell-complex plus some extra edges added to the faces of the cell-complex. We show that the number of additional edges does not exceed three times the number of faces in the cell-complex. Moreover, we show that for any path within the cell-complex there is a path on the skeleton that has just as much clearance.