On the Complexity of the Hypergreedy Matching Heuristic for the Euclidean Points in the Plane

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Abstract

Let V be a set of Euclidean points in the plane where $n = |V|$ is an even number. A *perfect matching* of V is a set of edges such that each vertex of V is incident to exactly one edge. An *optimal perfect matching* of V is a perfect matching with minimum total edge weight. The ***hypergreedy*** heuristic for perfect matching due to Plaisted runs in $O(n^2 logn)$ time, for a complete edge weighted graph satisfying the triangle inequality, and obtains an approximate solution with weight bounded above by $2log_3n - 1$ times the optimal weight. We show that this heuristic can be implemented in $O(nlog^2n)$ time for Euclidean points in the plane. In particular if the input is a convex polygon, we can implement the algorithm in $O(nlogn)$ time. For a given natural number $t < log_3n$ the ***t-basic graph*** is a collection of sparse connected components selected from the complete graph formed by V. We denote by ***even*** and ***odd*** connected components with an even and an odd number of vertices, respectively. The total edge weight of the ***t-basic graph*** does not exceed, for a given t, $2t$ times the weight of the optimal perfect matching of V. The ***hypergreedy*** constructs the ***t-basic graph*** for $t = \lfloor log_3n \rfloor$. We show that for $t < log_3n$ we can provide an approximate solution bounded above $2t + 1$ times the optimal solution, and it can be implemented in $O(max\{n^2, (\frac{n}{3^t})^3\})$ time.

The ***t-basic graph*** is constructed recursively from the *(t-1)-basic graph.* The ***1-basic graph***, is simply the nearest neighbor graph. It consists of ***odd*** and ***even*** connected components. We represent each connected component by a cluster of Voronoi regions of the vertices in the component. Next, the ***auxiliary graph*** is constructed, where each two adjacent connected components, treated as units, are joined by the shortest edge between them. To get the 2-basic graph from the 1-basic graph, we find for each odd unit its nearest odd unit.

In general, when the *i*-basic graph is obtained from the *(i-1)*-basic graph, we repeat the same process, which takes $O(nlogn)$ time. Hence time complexity for constructing the *t*-basic graph requires $O(tnlogn)$ time. For $t = \lfloor log_3n \rfloor$, the t-basic graph is guaranteed to contain only even connected components, therefore the ***hypergreedy*** spends $O(nlog^2n)$ time constructing it. Each such even component is duplicated and we extract from it a perfect matching, in a linear time. Thus the overall time complexity of the ***hypergreedy*** for the Euclidean points in the plane is $O(nlog^2n)$. The t-basic graph of a convex polygon can be found in $O(tn)$ time, and the ***hypergreedy*** can be implemented, for such input, in $O(nlogn)$ time.