Clustering/Hashing Points in the Plane with Maxmin Criteria

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Abstract

This paper presents efficient algorithms for several problems related to clustering and hashing points in the plane. One of the problems is to find an angle of projection to maximize the minimum relative gap among projected points. Another problem is to maximize the maximum relative gap, where 'relative' means that the gap is divided by the width of the point set, which is defined to be the distance between the farthest projected points.

The best known algorithm for the first problem runs in $O(n^2 \log n)$ time using quadratic space. In this paper we present a linear-space algorithm based on plane-sweep on the line arrangement in the dual plane. The algorithm runs in $O(n^2 \alpha(n) \log^2 n)$ time, where $\alpha(n)$ is the functional inverse of the Ackerman function. This is less efficient in time, but much more efficient in space.

The second problem is almost the same as the well-known problem of finding the maximum-area triangle for a set of points, which is known to be solved in $O(n^2)$ time using linear space based on topological sweep. The only difference is that in our case each gap divided by the width of the set at each angle must be evaluated.