IMPROPER INTERSECTION OF ALGEBRAIC CURVES

Shreeram S. Abhyankar*, Srinivasan Chandrasekar**, and Vijaya Chandru**

* Departments of Mathematics, Computer Science & Industrial Engineering

**School of Industrial Engineering, Purdue University, West Lafayette, Indiana 47907, U.S.A.

Abstract

Algebraic space curves are widely used in computer aided geometric design. These curves include Hermite interpolants, splines of various kinds and those arising from intersections of two or more algebraic surfaces. Two interrelated topics involving algebraic space curves that are of both mathematical and computational interest are *representation* and *intersection*. Intersection problems for algebraic plane curves have been elegantly solved using classical algebrogeometric techniques. These successes may be viewed as straightforward consequences of Bezout's theorem applied to the "proper" intersection of algebraic plane curves. This theorem implies that two algebraic curves of degree m and n intersect in no more than m n points on a plane, unless they overlap. This, in general, provides the least upper bound for plane curves. Such well defined bounds are hard to derive for curves in 3-dimensional space, as Bezout's theorem does not extend to such "improper" intersections.

In this paper we consider the intersection of algebraic curves in 3-dimensional space. We obtain an upper bound on the number of intersection points. The bound is only a function of the degrees of the individual curves. An important step in the proof is the construction of surfaces which completely contains one of the curves, but has only a finite number of points of the other. A number of special cases of the intersection problem for low degree curves are studied in some detail. Many of the ideas used in the proofs are algorithmic in nature and thus can be useful in the explicit computation of the intersection points. Extensions of the approach to intersection problems in k-space will be discussed.