Shoving a Table Into a Corner

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Abstract

In the Euclidean plane, consider a simple polygon P with N vertices and a wedge W with fixed angle ω about its apex O, $\omega \leq \pi$. Let $d(x, y)$ denote the distance between two points x and y. We consider the following placement problems:

Problem CWP (Closest Wedge Problem): Determine the position of P, with translations and rotations allowed, such that $\min\{d(O,p) : p \in P\}$ is minimized and such that P is in the interior of W. Determine also the point of P that realizes the minimal distance.

Problem CWP-v: Similar to CWP but $\min\{d(O,v) : v \text{ vertex of } P\}$ is minimized.

These problems are also relevant to compliant motion planning and can also be thought of as a type of guarding problems: if the wedge is considered as the field of view of some camera or guard. We consider the equivalent problems of holding P fixed and rotating W. We solve the problems in $O(N \log N)$ time, and in $O(N)$ time if P is convex or if we are given an appropriate Voronoi diagram.

We define a *placement* of O as a given position of O such that P is supported by each of the half-lines defining the W. A *stable* placement will be such that an edge of the convex hull of P lies on one half-line of W and an *optimal* placement is one that solves the CWP or the CWP-v problem. Let also *Cloud(P,ω)* be the set of all possible placements of O. We show that it is a chain of circular arcs around the polygon constructible in linear time. To solve problem CWP, we traverse Cloud(P,ω) to obtain its intersection with the generalized Voronoi diagram of the set of edges and vertices of P. For problem CWP-v we use the Voronoi diagram of the set of vertices. We then look at the distances between each arc segment and the edge (vertex) corresponding to the Voronoi region in which the arc segment is contained. In linear time we find the position of O that minimizes the distance between O and P (or O and vertices of P), and hence the optimal placement. For P convex, we can reduce the time to $O(N)$ since the generalized Voronoi diagram of the exterior can be obtained in time proportional to N. We obtain a similar result for problem CWP-v, by noting that only the Voronoi diagram in the exterior of P is needed.