

Clamping a Polygon

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Abstract

A clamp is composed of two opposing parallel edges with variable distance between them. We present a model that describes when the edges of the clamp and some edges of a polygon produce a *secure clamp* on the polygon. Although we cannot exhibit a polygon for which a secure clamp does not exist, we can only prove that secure clamps always exist for convex polygons and for certain polygons composed of two convex chains. The proof techniques are difficult and it is not obvious how to generalize them.

The following properties are desirable when a polygon is clamped securely: (1) the polygon, once clamped, cannot rotate in the plane, although it may be able to slide along the grippers without twisting; (2) if the clamp is mispositioned slightly with respect to the polygon, as the grippers close together they will shift the polygon but not lose touch with it entirely. Neither of these properties can be obtained in general without some friction between the grippers and the object, so we also imagine that the hand is made of firm rubber (like caliper brakes on a bicycle).

In the full paper we define formally what constitutes a secure clamp. We state the requirements here informally: (1) both grippers must contact the object in at least one point; (2) at least one gripper must contact the object in two points; (3) the foot of the perpendicular from the point in (1) to the other gripper must lie between the points in (2); and (4) at each of these contact points, the interior of the polygon lies between the grippers. Figures 1 and 2 show some examples of secure and insecure clamps. This purely geometric definition considers neither friction nor any of the moments of the polygon being clamped. While it may not be completely realistic from a mechanical standpoint, the model nonetheless provides conditions that seem necessary to clamp an object securely in the real world, and is justified intuitively by the experience of anyone who has used a vise.

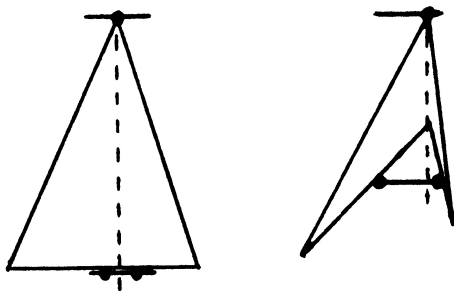


Figure 1. In these drawings of two secure clamps, the dashed line shows the foot of the perpendicular from the contact point on the top gripper.

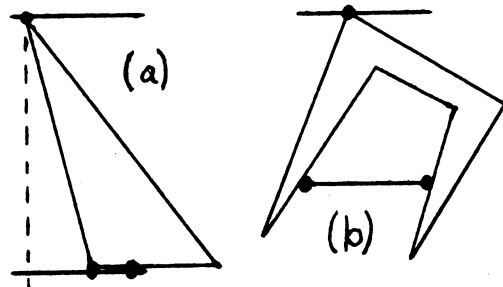


Figure 2. Neither of these clamps is secure. The clamp in (a) violates condition (3), while the clamp in (b) violates condition (4) in the definition.

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