# An Average Running Time Analysis of a Backtracking Algorithm to Calculate the Measure of the Union of Hyperrectangles in $d$ Dimensions

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## Abstract

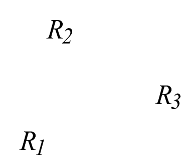
Problem MEASURE is the problem to calculate the measure of the union of $n$ hyperrectangles $R_i = [b_{i1}, e_{i1}] \times \cdots \times [b_{id}, e_{id}], i = 1, \ldots, n$, in $d \geq 2$ dimensions, $|\cup_{i=1}^{n} R_i|$, where $b_{ij}$ and $e_{ij}$ are numbers such that $b_{ij} \leq e_{ij}$ [3]. An $O(n^{d-1} \ln n)$ worst-case time algorithm to solve MEASURE for $d \geq 2$ [1] and an $O(n^{d-1})$ worst-case time algorithm for $d \geq 3$ [2] are known. In this paper, we propose a backtracking algorithm to solve MEASURE, analyze its average running time ((4) to (7) in Section 5), and show that the backtracking algorithm is more efficient than the former algorithms[1, 2] when $d \geq 3$ and there are many large hyperrectangles $R_i$ ((8) and (9) in Section 6).


## 1 Introduction

The problem MEASURE is defined as follows[1, 2, 3]:

**Definition** $n$ hyperrectangles $R_i = [b_{i1}, e_{i1}] \times \cdots \times [b_{id}, e_{id}], i = 1, \ldots, n$, in $d \geq 2$ dimensions are given, where $b_{ij}$ and $e_{ij}, i = 1, \ldots, n, j = 1, \ldots, d$, are numbers such that $b_{ij} \leq e_{ij}$. Calculate the measure of the union of those hyperrectangles, $|\cup_{i=1}^{n} R_i| = |\{(x_1, \ldots, x_d) \mid \exists i (1 \leq i \leq n), (x_1, \ldots, x_d) \in R_i\}|$. $\Box$

**Example** When 2-dimensional hyperrectangles (i.e., rectangles) $R_1 = [0, 3] \times [0, 4]$, $R_2 = [0, 5] \times [1, 8]$ and $R_3 = [4, 8] \times [0, 8]$ are given, $|R_1 \cup R_2 \cup R_3| = 63$ (Figure 1). $\Box$




For MEASURE, Bentley[1] gave an algorithm that can solve the problem for $d \geq 2$ in the worst-case time $O(n^{d-1} \ln n)$, and Leeuwen and Wood[2] did an algorithm for $d \geq 3$ in the worst-case time $O(n^{d-1})$. In the paper, ln and lg denote $\log_e$ and $\log_2$, respectively.

By the way, MEASURE can be considered a generalization of the problem to count the number of unsatisfying assignments of the satisfiability problem SAT, which is denoted by COUNT-SAT in this paper. For example, the COUNT-SAT problem to count the number (denoted by $N'$) of unsatisfying assignments for the DNF equation

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