

Shapes of the Future

Some Unsolved Problems in High-Dimensional Intuitive Geometry

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By "intuitive geometry" I mean the geometry of subsets of Euclidean n -space such as spheres, cubes, and simplices – objects which are so simple and seemingly well understood that all natural and interesting questions about them should be easy to answer. Focusing on such "intuitive" objects, this lecture will present a number of easily stated problems that remain unsolved despite the efforts of many mathematicians. Let us say that a problem is "essentially k -dimensional" if its k -dimensional version is open but the problem has been settled in all lower dimensions. I'll present essentially k -dimensional unsolved problems for $k = 2, 3, 4, 5, 6, 7, 21, 28,$ and 427 , and say something about the background of each. The problems have been chosen for their intrinsic mathematical interest, and in some cases also for their relationship to areas of application (error-correcting codes, design of experiments, linear programming). In some cases there are relevant background results concerning computational complexity, and in other cases there are reasons for believing that experimental computation could lead to useful insight.